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# BUREAU OF STANDARDS JOURNAL OF RESEARCH

Vol. 3 AUGUST, 1929 No. 2

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# EFFICIENCY OF MACHINISTS' VISES

By H. L. Whittemore and L. R. Sweetman

ABSTRACT

Although the industrial arts rest fundamentally on the simple hand tools which have been known and used for centuries, no tests have been reported previously showing the most efficient vise for a given job. This investigation was undertaken to determine the relationship between the size of the vise and its efficiency by performing typical shop operations on material held in the vise. These operations consisted of sawing, bending, and riveting steel specimens and were carried out under carefully standardized conditions, using 12 vises of the stationary bottom type having different lengths of jaw from 2 inches (9 pounds) to 9 inches (282 pounds).

In those tests which may be described as static tests (for example, the sawing tests) in which the movement of the tool was large compared with the movement of the vise, the tests showed no appreciable difference in the efficiency with which the work was performed.

On the contrary, in the dynamic tests (for example, the riveting tests and some of the bending tests on large specimens) the weight, or inertia, of the vise had an appreciable effect on the efficiency with which the work was performed. The efficiency of the 5½-inch (102-pound) vise was greater than for any of the lighter vises but was about the same for all vises larger than 5½ inches (102 pounds).

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## I. INTRODUCTION

In this age of mass production of accurate machines, such as automobiles, using intricate automatic machine tools, we often lose sight of the fact that our industrial arts rest fundamentally on the

simple hand tools which have been known and used for centuries. Before there were metal cutting machine tools a casting was finished by chipping the surface using a cold chisel and hand hammer. As the piece neared the finished size hand files were used to produce a smooth surface, followed by a hand scraper if an accurate surface was required.

Unless the work is very large, vises are necessarily used to hold it in the position desired by the workman. Because there is nothing spectacular or impressive about machinists' vises, we lose sight of the fact that these gripping devices are a most essential element of our mechanical equipment.

Although some vise manufacturers have based their designs on the results of strength tests, until recently no investigation has been undertaken to determine the most efficient vise for a given job. It is evidently absurd to select the smallest vise in which the piece can be held. Whatever the operation to be performed, we know that it consists in applying forces tending to move the piece, and that these forces are resisted by the vise and the workbench to which it is bolted. It is convenient to divide the forces into static and dynamic; that is, those exerted by slowly moving bodies, such as files, which exert a more or less uniform force to the piece and those exerted by rapidly moving bodies, such as hammers, which exert great force for short periods of time.

From experience with machines it is evident that under static forces the size and weight of the vise is of comparatively little importance because the forces acting on the piece are transmitted through the vise to the bench and the floor. On the contrary, impact or dynamic forces are largely expended in moving the piece and the vise through short distances. If a larger vise is used, its greater inertia makes this distance less and decreases the portion of the energy of the blow which is expended in moving the work and the piece. Evidently under impact, the size of the vise will have an effect on the efficiency with which the work is performed. Everyone knows that the energy for vise work is the muscular energy of the workman. As this is probably the most expensive energy used in mechanical processes it is of great economic importance to conserve it.

The investigation described here on typical shop operations was undertaken by the Bureau of Standards to determine the relationship between the size of the vise and the efficiency with which the work was performed. These operations, each carried out under carefully standardized conditions, were sawing with a hack saw and bending and riveting under impact.

## II. THE VISES AND BENCH

### 1. VISES

Twelve new vises of the stationary bottom type, typical of the vises used in this country, were used. No vises having swivel bases were used in these tests. They ranged in size from the smallest, having jaws 2 inches long (9 pounds), to the largest, having 9 inches (282

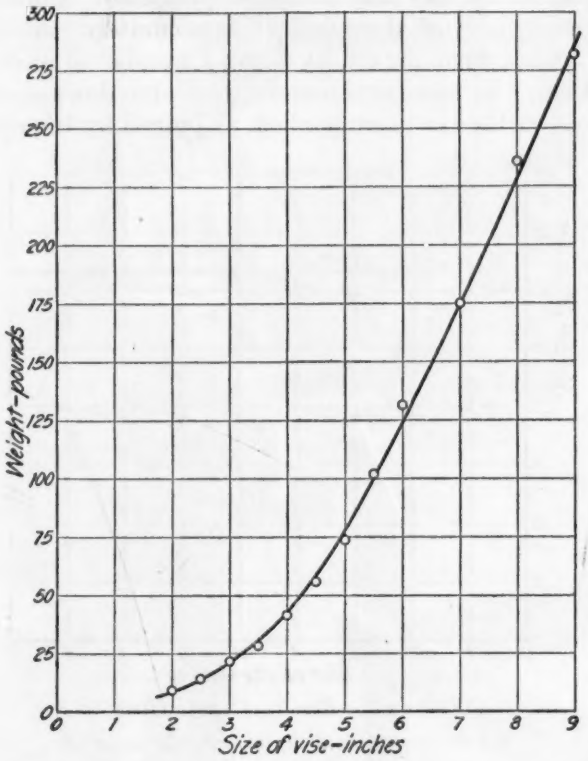


FIGURE 1.—*Relation between the size (length of jaw) and the weight of the vises*

pounds). The data for these 12 vises are shown graphically in Figure 1.

### 2. BENCH

Each vise was bolted to the bench shown in Figure 2, which consisted of a wooden frame, bolted together, and a top made from 2-inch planks. It was, probably, as rigid as the workbenches to which vises are attached in shops. The weight of the bench was 250 pounds.

After each vise was tested it was removed and another bolted to the bench in the same place.

## III. THE TESTS

### 1. SAWING TESTS

As representative of operations in which a more or less constant force is applied to work—such, for example, as filing or sawing—sawing with a hack saw was chosen.

Rather elaborate apparatus would be required to measure the forces acting on the saw and the effect produced. Consequently, a statistical study of work done under less accurately controlled conditions was made by cutting off, by hand, a number of pieces from the same steel bar. In these experiments, the same downward and horizontal forces on the saw were applied, as judged by the operator.

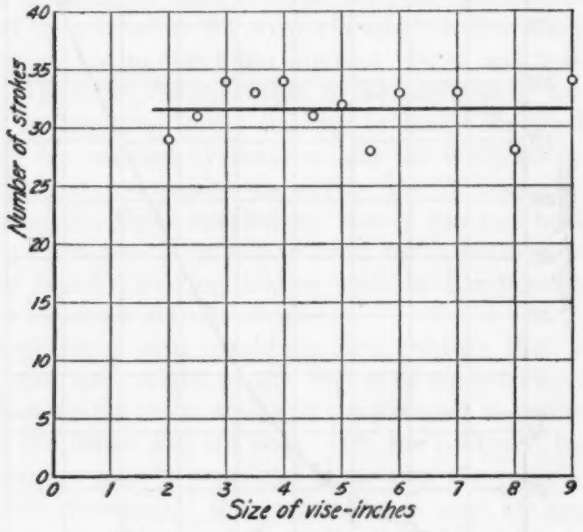


FIGURE 3.—*Results of sawing tests*

The efficiency was practically the same for all sizes of vise

A 12-inch hack-saw frame and 12-inch high-carbon tool-steel saw having 14 teeth per inch were used for all these tests.

The specimen was held horizontally at the middle of the faces of the jaws and the cut made ½-inch from the jaws, as shown in Figure 2.

Using a new blade, three cuts were made in each vise. The number of strokes required to sever the specimen was recorded, and the average for each vise is given in Figure 3. These results show that the differences were, in all probability, due to unavoidable variations in the downward force on the saw. There is no consistent difference which can be attributed to the size of the vise.

It is evident that for the conditions used in the sawing tests there is no difference in the efficiency with which filing, sawing, or similar operations are performed in vises of different sizes.

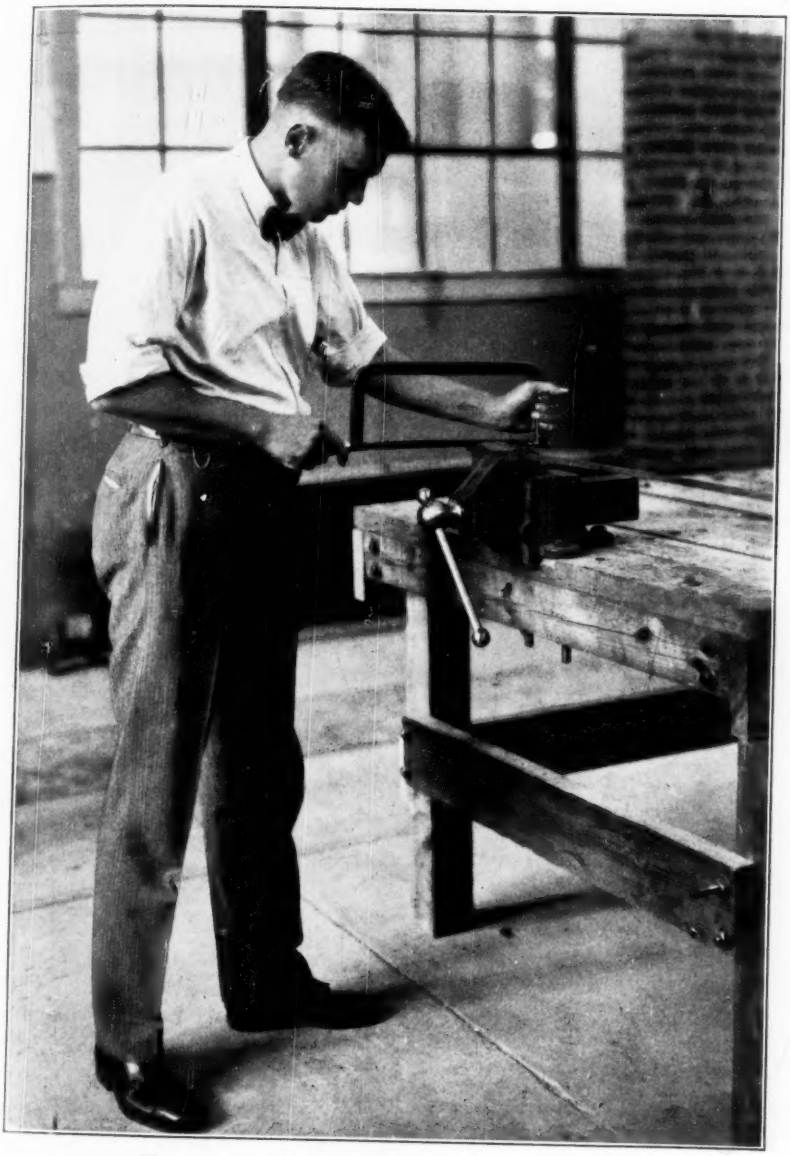

FIGURE 2.—*Method of making the sawing tests*

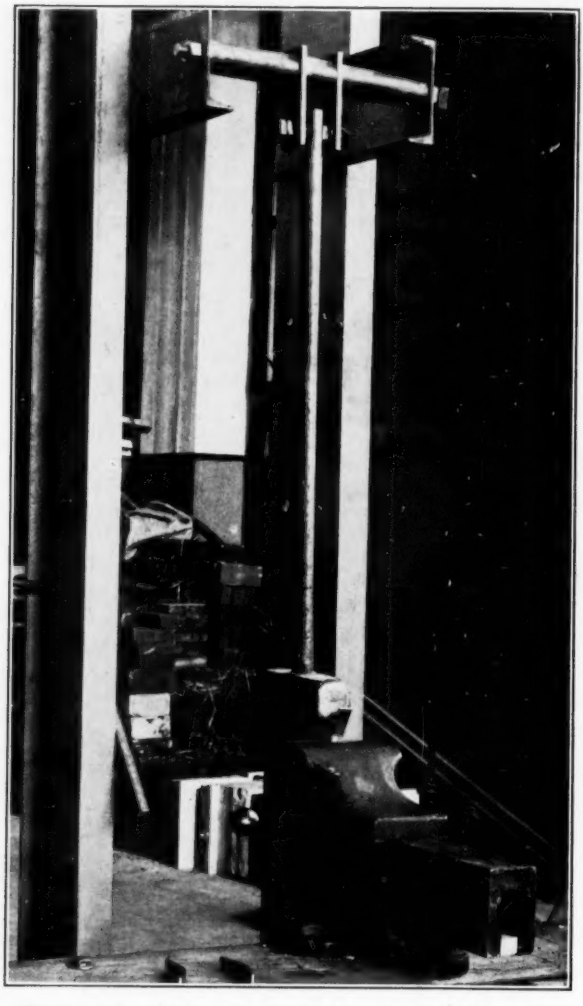

FIGURE 4.—*Apparatus for making the bending tests*

The specimen was held vertically in the vise and bent by the sledge swinging downward when released from a given height. Specimens after testing are shown on the bench.



UNIV OF MICH

## 2. BENDING TESTS

To simulate the bending of material held in a vise by horizontal blows of a hand hammer, an apparatus shown in Figure 4 was used. Hammer blows of known energy struck similar specimens held in the jaws of the vise. The angle through which the specimen was bent was taken as a measure of the useful work performed by the blow and served as a basis for comparing the efficiencies of the different vises.

A sledge weighing 12 pounds was swung on an axle carried by the vertical frame which allowed both horizontal and vertical adjustment of the sledge with respect to the vise.

The specimens were held vertically in the vise and bent by one or more blows of the sledge.

(*a*) First Series.—To determine the height from which the sledge should be released preliminary tests were made. Specimens were held in a vise and struck horizontally with a 2-pound hand hammer. A blow was used which, it is believed, was approximately such a blow as a workman would deliver if he were employed continuously on similar work.

Repeated tests of specimens bent with the hand hammer and others bent with the sledge showed that if the sledge was raised 8½ inches above its lowest position and allowed to swing downward and strike the specimen once at the lowest position, that the specimen was bent through approximately the same angle as it was bent by one blow of the hand hammer. This fall of 8½ inches (energy 102 in.-lb.) was therefore used in the first series of bending tests to simulate the blow which a workman would ordinarily strike with a hand hammer.

The specimens, ⅛ inch thick, ¾ inch wide, and 4 inches long, cut from hot-rolled mild steel bars, were held vertically in the vise, so that the upper end was 2⅛ inches above the jaws.

Specimens of this size were bent through a comparatively large angle, and any differences due to the size of the vise could be measured readily.

The axle was adjusted for each vise, so that the center of the face of the sledge when in its lowest position was 2¼ inches above the jaws and just touched the ¾-inch face of the specimen.

The bending tests were made by bolting the vise to the bench, securing the specimen in the jaws, drawing back the sledge and releasing it. The deformed specimen was removed from the vise and the angle measured. Twelve specimens (one for each vise) were cut from each of three similar bars. The results from the three specimens tested in each vise were averaged. In this way the effect of variations in the properties of the specimens was minimized.

The average of the results on the three bend specimens for each vise are shown in Figure 5. For one vise only did the individual values

differ by as much as 1° from the average. It is believed, therefore, that this bending apparatus gave reproducible results, and that three specimens for each vise were sufficient.

(*b*) SECOND SERIES.—Although the size of the vise had no appreciable effect on the results of the first series, it was felt that differences might be found if the sledge had been raised to a different height.

Additional tests were therefore made, raising the sledge 3, 5, 7, 9, and 11 inches, and allowing it to strike the specimen once. Three specimens, similar to those previously used and all cut from the same

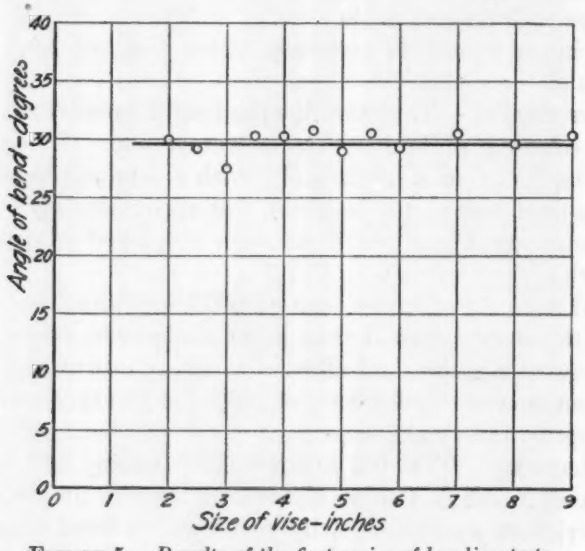


FIGURE 5.—*Results of the first series of bending tests*

Sledge raised 8½ inches. The efficiency was practically the same for all sizes of vise.

bar, were tested for each height. For these tests only the 2-inch (9-pound), 5½-inch (102-pound), and the 9-inch (282-pound) vises were used. The results are shown in Figure 6.

The angle of bend is directly proportional to the height to which the sledge was raised; the deformation was therefore directly proportional to the energy of the blow. It is evident from Figure 6 that the results for the three vises are the same within experimental error.

The results of the first and second series of bending tests, therefore, show that the size of the vise in which the work is held has no appreciable effect on the efficiency with which material is bent by one blow (energy 102 in.-lb.) simulating that from a hand hammer.

(*c*) THIRD SERIES.—As a workman may use a sledge requiring the use of both hands for some work, a third series of bending tests was made allowing the sledge to fall from a height of 27 inches and delivering a blow having an energy of 324 in.-lb. Specimens having

a larger cross section than in the first and second series were used for this series. They were soft steel ⅜ inch thick and 1½ inches wide and were held vertically in the vise as for the previous tests. As one blow did not bend the specimen through a large angle, each specimen was struck seven blows. Then the angle was measured. Three specimens were tested in each vise. The average angle for the three specimens tested in each vise is given in Figure 7.

The results of this series show that for heavy bending work requiring the use of a sledge there is a great increase in efficiency up to a certain size, if the work is held in a heavy vise.

The specimens, held in vises having 7-inch (175-pound), 8-inch (236-pound), and 9-inch (282-pound) jaws, gave on the average

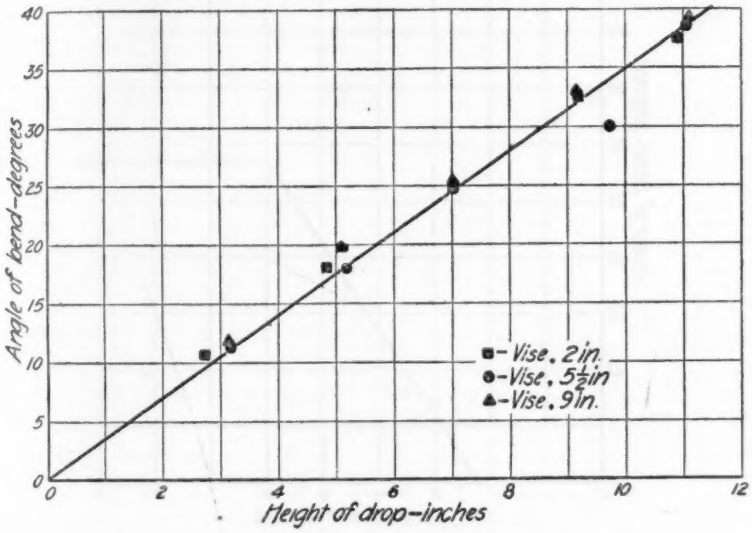


FIGURE 6.—*Results of second series of bending tests*

Sledge raised to different heights. The angle of bend is proportional to the height to which the sledge was raised.

about the same results, showing that there is no measurable advantage in using vises larger than 7-inch (175-pound) when heavy blows are used to perform the work.

### 3. RIVETING TESTS

(*a*) FIRST SERIES.—To determine the effect of the size of the vise when upsetting or riveting soft steel, specimens 3/32 inch in diameter and 1 3/16 inches long were cut from the same bar and the effect of axial blows observed by measuring the shortening of the specimen. To obtain deformations which could be measured readily, five blows were used for each of the riveting specimens in the first, second, and third series of riveting tests.

To avoid slipping of the specimen in the jaws, the steel holder shown in Figure 8 was used. It was held in the jaws with the hole for the specimen vertical and with the flange resting on the tops of the jaws.

The specimens were placed in the holder, projecting 5/16 inch and the sledge, used for the bending tests, adjusted to drop vertically on the upper end of the specimen. The axle was adjusted for each vise, so that the handle of the sledge was horizontal and the sledge struck the specimen in the middle of the striking face.

It was found, experimentally, that this 12-pound sledge, dropping 10 inches and striking a blow having an energy of 120 in.-lb. caused

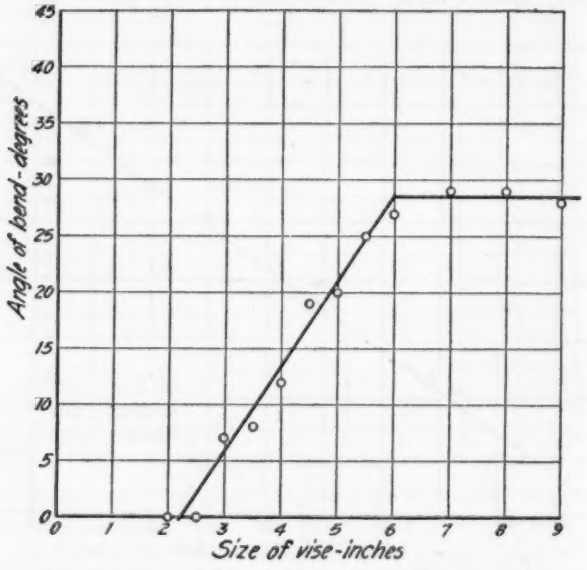


FIGURE 7.—*Results of third series of bending tests*

Sledge raised 27 inches. The efficiency increased greatly as larger vises were used up to 5½ inches. There was no increase in efficiency for vises larger than 5½ inches.

about the same shortening of the riveting specimens as a blow having energy comparable with that of a 2-pound hand hammer.

The tests were made by supporting the sledge 10 inches above the specimen by a stick, as show in Figure 9. The stick was withdrawn quickly, allowing the sledge to drop on the specimen. This operation was repeated five times, then the specimen was removed from the holder and its length measured. Three specimens were tested in each vise, and the average results are shown in Figure 10.

These results show that the shortening was greater the larger the vise up to the 5½-inch (102-pound) vise. There was no appreciable increase in the shortening for the larger vises. The specimens tested

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FIGURE 9.—*Apparatus for making riveting tests*

The sledge was supported at the desired height by a stick which was withdrawn quickly allowing the sledge to drop on the specimen.

in the 5½-inch (102-pound) vise shortened about three times as much as those tested in the 2-inch (9-pound) vise.

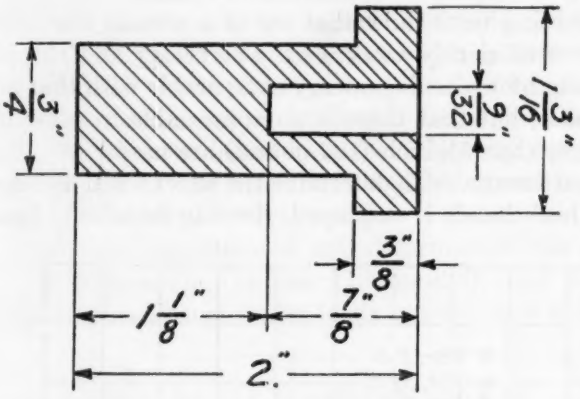


FIGURE 8.—*Holder for riveting specimens*

The holder was held vertically in the vise with the flange resting on the jaws. The specimens were placed in the recess in the holder.

(*b*) SECOND SERIES.—To determine the effect of varying the energy of the blow by varying the height of drop, riveting tests were also made, using drops of 4, 6, 8, 10, and 12 inches. For these tests only

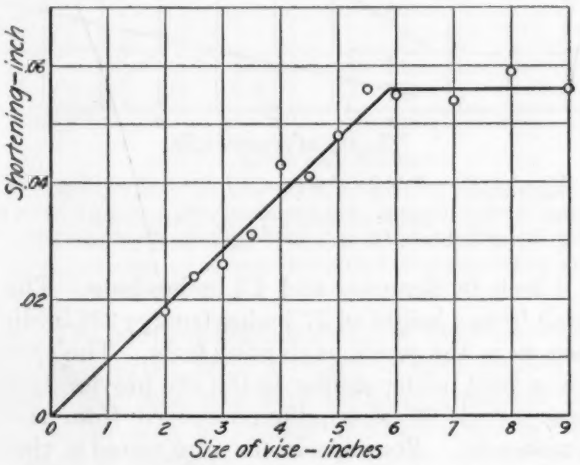


FIGURE 10.—*Results of the first series of riveting tests*

Sledge raised 10 inches. The efficiency increased greatly as larger vises were used up to 5½ inches. There was no increase in efficiency for vises larger than 5½ inches.

the 2-inch (9-pound), 5½-inch (102-pound), and 9-inch (282-pound) vises were used.

The results are shown in Figure 11. They show that the deformation increased as the height was increased, and also that the defor-

mation is greater for the 5½-inch (102-pound) vise than for the 2-inch (9-pound) vise, but that the 5½-inch (102-pound) and the 9-inch (282-pound) vises give about the same results.

These riveting tests show that up to a certain size the larger the vise the more efficiently work, such as riveting and chipping, is performed, using blows having energy comparable with that of a 2-pound hand hammer, but that there is no appreciable increase in efficiency if vises larger than 5½-inch (102-pound) are used.

(*c*) THIRD SERIES.—To determine the effect when a sledge requiring the use of both hands is employed, riveting tests were made on large

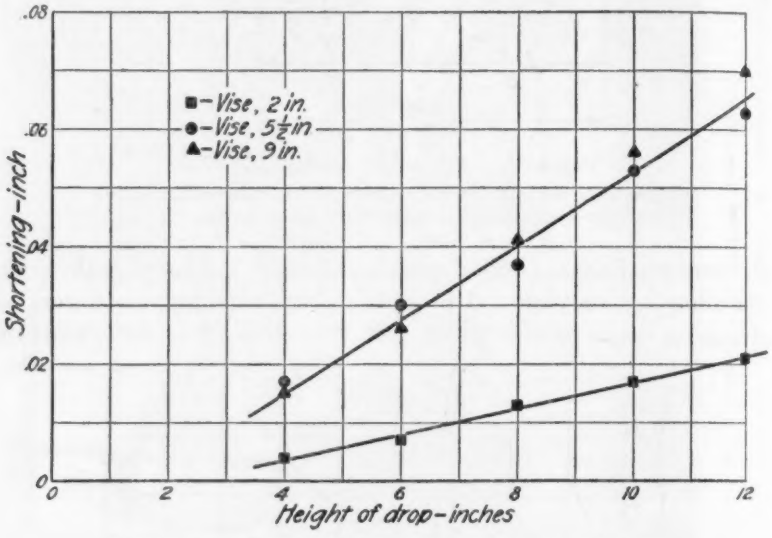


FIGURE 11.—*Results of the second series of riveting tests*

Sledge raised to different heights. The deformation of the specimens is greater for 5½-inch vise, than for the 2-inch vice, but the 5½-inch and 9-inch vises give about the same results.

specimens ⅝ inch in diameter and 1⅞ inches long. The sledge was allowed to fall from a height of 27 inches (energy 324 in.-lb.) and strike the specimen as in the previous riveting tests. The specimens were supported in a steel holder similar to the one previously used. Each specimen was struck 60 blows, then removed from the holder and the length measured. Four specimens were tested in the 6-inch (132-pound) vise and six in the 8-inch (236-pound) vise. Three specimens were tested in each of the other vises used. The average results are shown in Figure 12.

This series showed that there is an increase in efficiency as the size of the vise increases up to 5½ inches (102 pounds). For larger vises there is no appreciable increase in the efficiency even with these heavy blows having an energy of 324 in.-lb.

## IV. DISCUSSION

In these tests the vises were tightly bolted to a bench having approximately the rigidity of workbenches usually used in a shop. If the bench had been much less rigid, these tests would not have simulated reasonably good shop conditions. It is believed that for the sawing tests and the first and second series of bending tests the rigidity of the bench was not an important factor, because the forces applied to the specimens acted through relatively long distances, and the deformation of the bench was negligible compared with the distances through which the forces acted.

To determine the magnitude of this deformation, the movements of the 2-inch (9-pound), 5½-inch (102-pound), and 9-inch (282-pound) vises were measured under loads applied to a specimen held

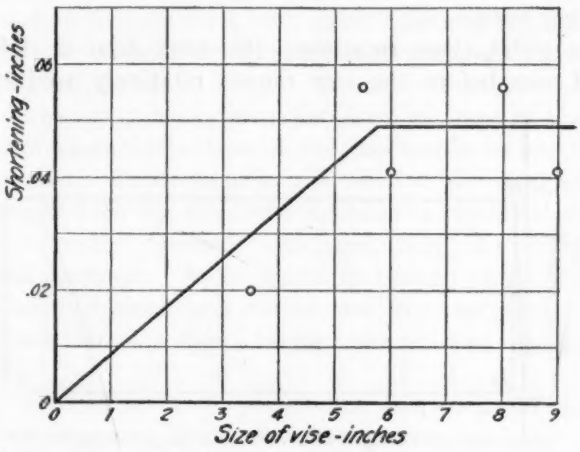


FIGURE 12.—*Results of the third series of riveting tests*

Sledge raised 27 inches. The efficiency increased as larger vises were used up to 5½ inches. There was no appreciable increase for vises larger than 5½ inches, even with these heavy blows having an energy of 324 in.-lb.

in the vise. The vertical displacements of the vises were measured, using a dial micrometer, under a 50-pound load applied by a weight. The horizontal displacements were similarly measured under a horizontal load of 50 pounds applied through a spring balance. The results are given in Table 1.

TABLE 1.—*Movement of vises*

| Size of vise (in inches) | Movement | |
|---|---|---|
| | Vertical force, 50-pound | Horizontal force, 50-pound |
| | *Inch* | *Inch* |
| 2 | 0.005 | 0.040 |
| 5½ | .003 | .022 |
| 9 | .004 | .048 |

The movement, of course, depends somewhat upon the forces exerted by the bolts used to attach the vise to the bench, but as these bolts were tightened until the washers under the nuts crushed into the wood it is believed that these forces were as high as good shop practice permits. The movement of the vises is probably caused, at least in large part, by the deformation of the bench under the load.

Considering the sawing tests, we may draw the hypothetical work diagram shown in Figure 13, in which the ordinates *BD* represent the horizontal forward force parallel to the blade and *OE* the length of the cutting stroke (11½ inches). When a cut is started, the force increases as shown by *OB* and is then constant for the remainder of the stroke. The deformation of the bench and vise are represented by the distance *OD*, which is less than 0.05 inch for a force of 50 pounds. It would be much less, about 0.01 inch, for the force used in the sawing tests.

The area *OBD*, then, represents the work done in deforming the bench and vise before the saw moves relatively to the specimen.

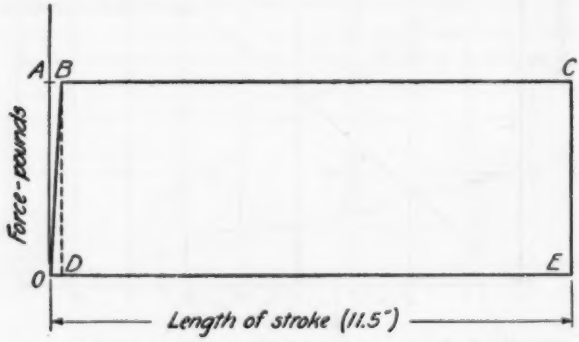


FIGURE 13.—*Work diagram for sawing tests*

The area *OBD* represents the work required to deform the vise and bench. The area *DBCED* represents the work expended in cutting the specimen.

The area *DBCED* measured the work done in cutting the specimen. Assuming that *OD* is 0.01 inch and *OE* 11½ inches, the work lost due to the deformation of the bench and vise is 0.04 of 1 per cent of the total work, which is inappreciable.

The distance through which the force acted in the first and second series of bending tests was about ¾ inch. This, again, is large compared with the movement of the vise. As the variation of the force as the specimen bends is unknown, no attempt will be made to estimate the loss but, as in the sawing tests, it, in all probability, is negligible.

It should be noted that in both the sawing tests and in the first and second series of bending tests the forces acted for an appreciable length of time. For this reason the inertia of the vise has little

effect and the vise moves until the bench exerts forces sufficiently great to counteract the forces exerted on the specimen. This conclusion is justified by the fact that the size of the vise had no appreciable effect on the efficiency with which sawing or bending was performed.

In the third series of bending tests each specimen was struck seven blows. As the specimens held in the two smallest sizes of vise were not bent, all of the energy must have been expended in elastic deformation of the specimen, vise, and bench.

Figure 7 shows, however, that whatever the magnitude of this elastic deformation, using a vise larger than 6 inches (132 pounds) does not increase the efficiency with which bending is performed with a sledge.

In the third series of bending tests and in all the riveting tests the forces on the specimen act for a very short time and their magnitudes are unknown. It is therefore impossible to estimate the movement of the vise in these tests.

We can, however, reason from the laws of impact that for the same energy of blow the motion of the vise would be less the lighter the hammer used. Consequently, the relative advantage of using a larger vise found with the 12-pound hammer is greater than could be expected if a smaller hammer was used, such, for example, as a 2-pound hand hammer. As no hammer heavier than 12 pounds is likely to be used on work in a machinists' vise, no greater efficiency can be expected from the use of larger vises than are shown by these tests.

The load required to cause permanent shortening of the riveting specimens was found to be about 4,000 pounds, and the top of the bench would have deflected considerably under a static load of this magnitude. It is evident, therefore, that in all the riveting tests the ineritia or "anvil effect" of the vise was much more important than the support offered by the bench. The results showed a great increase in the efficiency with which riveting was performed as the size of the vise was increased up to 5½ inches (102 pounds). The fact that no appreciable increase was shown for larger vises tends to show that for pieces which can be held in smaller sizes there is little advantage in using vises larger than 5½ inches (102 pounds) if the work is to be performed either with a hand hammer or a sledge.

## V. CONCLUSIONS

1. Tests made by sawing steel specimens held in vises of different sizes, from 2-inch (9-pound) to 9-inch (282-pound), showed no appreciable difference in the efficiency with which the work was performed.

2. Tests made on small steel specimens held in vises of different sizes, from 2-inch (9-pound) to 9-inch (282-pound), by blows having energy comparable with that delivered by a 2-pound hand hammer used under average working conditions, showed no appreciable difference in the efficiency with which the work was performed.

3. Tests made by bending large steel specimens held in vises of different sizes, from 2-inch (9-pound) to 9-inch (282-pound), by blows having energy comparable with that delivered by a 12-pound sledge used under average working conditions, showed an increase in the efficiency with which the work was performed as the size of the vise was increased up to 6 inches (132 pounds). The efficiency was about the same for all vises larger than 6 inches (132 pounds).

4. Tests made by upsetting or riveting small steel specimens held in vises of different sizes, from 2-inch (9-pound) to 9-inch (282-pound), by blows having energy comparable with that delivered by a 2-pound hand hammer used under average working conditions, showed that the efficiency using a 5½-inch (102-pound) vise was about three times that found for a 2-inch (9-pound) vise. The efficiency was about the same for all vises larger than 5½ inches (102 pounds).

5. Tests made by upsetting or riveting large steel specimens held in vises of different sizes, from 2-inch (9-pound) to 9-inch (282-pound), by blows having energy comparable with that delivered by a 12-pound sledge used under average working conditions, showed that the efficiency using a 5½-inch (102-pound) vise was greater than for any of the smaller vises. The efficiency was about the same for all vises larger than 5½ inches (102 pounds).

WASHINGTON, January 9, 1929.

# EFFECT OF SERVICE ON THE ENDURANCE PROPERTIES OF RAIL STEELS

By John R. Freeman, jr., and Haig N. Solakian

ABSTRACT

A series of tests has been carried out to determine the effect of service on the endurance properties of rail steels. In a previous report data were given on the endurance properties of steel from new rails. In the present report data are given on the endurance properties of steel from rails from the same heats after service and from two rails which failed due to a transverse fissure.

It was found that the endurance properties of the steel were not affected by over 20,000,000 tons of traffic. In one group of rails very marked variation in results was found which was shown to be due to the presence of small internal cracks in the rails. Evidence was found which indicated that these rails probably contained small cracks before placing in service.

It was concluded that the service stresses imposed had not measurably affected the endurance properties of the steel, and, therefore, that the service stresses were less than the endurance limit of sound rail steel.

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## I. INTRODUCTION

Years of speculation on the causes of transverse-fissure failures in rails have resulted in two schools of thought. Both schools agree that the failure is progressive, starting from a nucleus like any

fatigue failure, and differing from most fatigue failures only in that the nucleus is inside the rail rather than at the surface. Other cases of internal nuclei for fatigue failures are, however, well known, as in the cases of ball bearings and of gear teeth.

Fatigue failures are due to the repeated stressing of the structure at stresses greater than the endurance limit of the material in that structure. The endurance limit may be defined as that minimum stress which, if repeatedly imposed, will cause cracks to develop which will progress under repetition of that stress, causing ultimate failure.

One school of thought takes the view that the stresses developed in rails under present-day wheel loads exceed the endurance limit of present-day rail steels and, consequently, no matter how sound and uniform present-day rail steels may be, all may ultimately fail through fatigue. If this view is correct, there are only two ways out of the dilemma—one to reduce the wheel loads, the other to change the type of steel used for rails, abandon the present type, and go to some type with an endurance limit greater than the imposed stresses. Since accurate knowledge of the stresses produced within the rail by the wheel loads is lacking, and since accurate knowledge of the endurance limit of rail steel under incompletely reversed stresses is also lacking, this view deserves consideration until it is definitely disproved. This school of thought might be termed the "stress school," as it holds that the nucleus for the transverse-fissure failure is created by the stress.

The other school of thought takes the view that if sound rail steel under present-day wheel loads is really subjected to repeated stress above its endurance limit, all rails in similar service would fail through transverse fissures after the same length of life. Since it is well known that this is not the case and that of two adjacent rails from the same heat, laid at the same time, one may fail from a transverse fissure and on examination may show other fissures in process of formation, while the other rail may show no sign whatever of any fissure, this school believes that the nucleus for a transverse-fissure failure is not created by imposed stress, but is preexistent in the steel before the rail is laid, in the form of some flaw. If the idea of the "preexistent nucleus" or "stress-raiser" school is correct, then neither the wheel loads nor the type of steel used for rails need be changed, if only some means can be found to produce the present type of steel free from the sort of flaws that act as nuclei for fatigue-fissure failures.

One difficulty in this theory is that it has not been definitely proved that nonmetallic inclusions (slag, "dirt," etc.) which are commonly agreed to act as stress raisers and to form nuclei for fatigue failures in many steel parts are present at the nuclei of transverse rail fissures, so that the exact nature of the nuclei is in doubt.

In a recent report [1] it was shown that the endurance limit in rotary bending of specimens cut from unused rails which met all specifications for static tests of rail steel varied from a maximum of 59,000 lbs./in.$^2$ to a minimum of 41,000 lbs./in.$^2$ If a larger number of samples were examined, this range might be widened. If the extreme endurance range be taken as $\pm 40{,}000$ to $\pm 60{,}000$ lbs./in.$^2$, it would appear that, if these values correctly represent the maximum and minimum endurance limits of ordinary rails, repeated rail stresses equivalent in severity to the stress range of 120,000 lbs./in.$^2$ ($\pm 60{,}000$) applied in testing would result in the failure of all rails by fatigue, and those equivalent to a stress range of 80,000 lbs./in.$^2$ ($\pm 40{,}000$) would result in no failures.

If, however, stress raisers that can act as nuclei for fatigue failure be present in the rail, but not in the particular specimens cut from it, the calculated applied load might be locally increased to a value above the endurance limit. Unless such stress raisers were met in the most highly stressed portions of the specimens examined, the endurance limit for the rail steel determined by the test would be above the endurance limit for the rail. It is not unreasonable to suppose that the true endurance limit for sound, clean, rail steel is well above 60,000 lbs./in.$^2$ and that the amount which actual tests indicate the observed endurance limit to drop below the true value is a measure of the effect of the stress raisers existing in the specimens used.

However, the wide range of endurance limits (40,000 to 60,000 lbs./in.$^2$) does not exist between specimens from comparable locations in the cross section of rails from the same heat in such tests so far made at the Bureau of Standards. Instead of a range of 20,000 lbs./in.$^2$, the range is reduced to 6,000 lbs./in.$^2$, and ordinarily to a much smaller range.

It would be possible to draw more exact conclusions on the endurance properties of rails were it feasible to use a whole rail, rather than specimens cut from it, for laboratory endurance tests. The construction and operation of an endurance-testing machine capable of applying and of measuring with sufficient accuracy the large loads required in the test of a full-sized rail would be so expensive that it is desirable first to secure as much information as possible from pilot tests of specimens cut from rails made with endurance machines now available.

Were it feasible to subject specimens cut from rails to a type of test in which a larger volume is subjected to the maximum stress than is the case with rotary-beam or rotary-cantilever testing machines, the sampling error would be diminished. Concurrently with the present investigation, the Bureau of Standards is studying the

[1] Freeman, J. R., jr., Dowdell, R. L., and Berry, W. J., Endurance and Other Properties of Rail Steel. B. S. Tech. Paper No. 368; 1928. This report is referred to throughout this paper as "the previous report."

axial loading, tension-compression endurance test, using a Haigh type of machine, with the expectation of applying that type of test to rail steel. Until that work is farther advanced, the use of such a test would involve uncertainties at least as great as those met in rotary-beam testing.

As was pointed out in the previous report, it appeared that a study of the endurance properties of rail steel before and after service might throw some light on the question as to whether or not service stresses in themselves create nuclei for transverse-fissure failure or whether the nuclei are preexistent. The ground for the hope that such tests would illumine the problem consists in the fact, brought out in the previous report, that, like other steels, rail steels may be strengthened by "understressing" in the endurance test and weakened by "overstressing."

All experimenters working on fatigue of metals agree as to the experimental facts that the fatigue resistance of a steel is increased when the specimen is repeatedly stressed at a stress below, but near to, its true endurance limit. The true endurance limit is found by applying repeated stress to various specimens, starting with a stress which will break the specimen in a relatively small number of cycles, and reducing the stress on successive specimens till a stress is reached at which the specimen will not break no matter how many stress cycles are applied. If such an unbroken specimen is then subjected to a higher stress, one, for example, that would break a fresh specimen in half a million cycles, the specimen that has been understressed will withstand many million cycles. It has been shown that understressing in the endurance test raises the static tensile strength if the understressed specimen is tested in tension.

It is not known how near the endurance limit the understress must lie to produce strengthening, how many cycles of stress must be applied, nor whether the specimen must be stressed in the same manner in the understressing and in the later endurance test. For example, it is not known whether understressing by axial loading will strengthen a specimen against higher stress applied by rotary bending, or whether it is only strengthened against higher stress applied in axial loading.

Students of fatigue also agree on the experimental fact that a specimen which has been repeatedly stressed at a stress above the endurance limit which, if applied a sufficient number of times, would break it but which has not yet produced a microscopically detectable crack, has been weakened by overstressing. Not very much is known about the quantitative effect of overstress. An overstress of 20 to 30 per cent above the true endurance limit, applied for a few thousand cycles, may reduce the apparent endurance limit for undamaged material quite appreciably, but it is not definitely known how various

stresses and various numbers of stress applications affect various steels.

While the theories to account for these experimental phenomena have been only tentatively put forth, a very reasonable point of view is that of Moore.[2] According to this theory, steel, not being homogenous but being made up of variously oriented crystals, contains tiny places less favorably oriented for withstanding a certain stress applied in a certain direction, and, under stress, the crystals at these locations undergo slip. Slippage in those places hardens and strengthens them much as slip, occurring generally throughout a metal, hardens it on cold working.

This slip so strengthens the unfavorably oriented places that they are able to transmit the load to surrounding metal without themselves undergoing further slip, so that, instead of incipient failure at those places, the result of the application of stress is a strengthening of the specimen. It is postulated that this slip merely moves the layers of atoms in some of the crystal slip planes without breaking down the cohesion between the layers of atoms. The layers are slid along, but not separated, and a crack does not result. The result is local cold working of tiny places within the metal, these places being automatically selected by the stress distribution within the metal. This conception is in accord with the observed phenomena of strengthening by understressing.

If the applied stress is too high, however, the unfavorably oriented areas become overworked before sufficient readjustment has taken place to distribute the load to surrounding material and slip and motion within the affected grain are so great that the layers are separated beyond the distance through which atomic cohesion can operate. In that case an incipient crack is formed which may at first be of submicroscopic dimensions and later grows into a visible crack. Instead of increased strength due to cold working, there is now a tiny flaw which acts as a "stress raiser," like any crack, so that damage, instead of being prevented by redistribution of the load, is concentrated because of increase in local intensity of stress. This conception agrees with the experimental facts of overstressing. This idea of the effect of overstress is somewhat analogous to that held by some students of transverse-fissure failures who postulate the development, under repeated stress, of "zones of exhausted ductility." The mental picture of these zones, however, calls for larger volumes of metal being affected than does that of submicroscopic incipient cracks.

Under repeated stress there are, then, two opposing factors, one tending to strengthen the material, one tending to weaken it. At

[1] Moore, H. F., Manual of Endurance of Metals Under Repeated Stress, Eng. Found. Pub. No. 13; 1927. What Happens When Metal Fails by "Fatigue," Trans. Am. Soc. Steel Treat., **9**, p. 539; 1926.

such a stress that the weakening tendency is so overcome by the strengthening tendency that no localities are stressed to the point where development of a real crack can occur, we have the true endurance limit. At any higher stress failure will ensue unless the metal has previously been improved by understressing.

Overstressing to the point where an actual crack starts should be quite readily detectable. Moore [3] says that once a fatigue crack is started it will spread under subsequent cycles of a stress much lower than the true endurance limit, and that relatively few cycles of overstress are required to start the crack. It would appear that the development of a crack or an incipient, submicroscopic separation, since it forms a stress raiser and intensifies the local stress, would more readily show its effect upon subsequent application of repeated stress, even though that stress were not applied in exactly the same direction as the stress causing the damage, than the development of tiny strengthened areas produced by a certain prior stress would show its effect on subsequent test.

From this point of view it appeared possible that the endurance testing of specimens from used rail and the comparison of the endurance limit of used-rail specimens from a given heat or ingot with the endurance limit of specimens from unused rail from the same heat or ingot ought to give a useful indication whether or not service had created overstressed volumes and incipient nuclei for fatigue failure throughout the rail. If rail steel that is originally sound and flawless has flaws created within it by overstress due to too high wheel loads, irrespective of the presence of stress raisers, such as inclusions and cracks in the steel before service, but due merely to the random orientation of the grains of the steel, then weakening of the steel due to overstressing should be readily apparent by a materially reduced endurance limit. Any microphotograph of rail steel will show such a number of differently oriented grains that grains unfavorably oriented to withstand a stress applied from any direction should be present throughout.

The effect of the application of stress below the endurance limit would be harder to detect, since it appears that many applications of understress, and of understress fairly close to the endurance limit at that, are required to produce appreciable change in properties, while fewer applications of overstress appear to be required to cause detectable damage. Stresses far below the endurance limit would not be expected to produce any detectable effect whatever.

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[3] Moore, H. F., and Kommers, J. B., The Fatigue of Metals, p. 227; 1927. Moore, H. F., Manual of Endurance of Metals Under Repeated Stress, p. 16; 1927.

The question whether excessive wheel loads above the endurance limit of sound rail steel are actually being applied in present-day practice is of fundamental importance in the transverse-fissure problem. If they are, the metallurgist must either provide a new type of steel with higher endurance limit or the railways must reduce the wheel loads. If they are not, and the evidence points to pre-existent nuclei due to the steel and not to the service, then a possible field for improvement of present-day types of rail steel presents itself if the source of the nuclei can be determined.

It was not expected that a complete and positive answer, such as might be gotten were it feasible to make endurance tests on whole rails under controlled conditions, would be obtained from the testing of random specimens taken from used rail with no record of the actual stresses that had been applied to that location within the rail represented by the breaking section of a specimen cut from the rail, since, beside the difficulties always met in sampling by which random samples may prove of better quality than the poorest part of the lot or object sampled, the cutting of the specimen from the rail would release internal stress that might be present in the rail itself.

As experience is accumulated in the detection of transverse fissures in the track itself by the Sperry rail flaw detector [4] or other devices and the progress of fissures in actual rail has been more closely followed, some of these questions may be answered on the basis of evidence so accumulated, from the point of view of actual service.

However, the importance of the transverse-fissure problem is so great that all possible lines of attack should be followed simultaneously. With due regard to its limitations, a study of the endurance properties of rail steel before and after service may be expected to throw some light on the fissure problem.

Through the cooperation of E. Stimson, chief engineer of maintenance of the Baltimore & Ohio Railroad and J. M. R. Fairbairn, chief engineer of the Canadian Pacific Railway, rails from some heats which had been tested before service were removed from track after approximately one and a half to two years' service and returned to the Bureau of Standards for a redetermination of the endurance limits of the steel.

## II. MATERIAL STUDIED

The record of the rails returned from service on the Baltimore & Ohio Railroad and Canadian Pacific Railway are given in Tables 1 and 2, respectively.

[4] Transverse-Fissure Detecting Devices, Am. Ry. Eng. Assoc., 28, p. 966; 1927.

TABLE 1.—*Record of rails returned from service on the Baltimore & Ohio Railroad*

| Railroad company No. | Manufacturers heat No. | Bureau of Standards heat No.[1] | Ingot No.[2] | Position in ingot of rail tested | | Location and service conditions as given by railroad |
|---|---|---|---|---|---|---|
| | | | | After service | Before service[2] | |
| L-15 | 203284 | 18 | 13 | BC[3] | AB[3] | Bridge 323; low rail 9° 2.5′ curve with superelevation of 5½ inches; 21,738,000 tons of traffic. |
| L-17 | 203284 | 18 | 15 | BC | AB | |
| S-13 | 203286 | 19 | 10 | BC | AB | Yellow dog tangent; 20,562,000 tons of traffic; S-13 in south and N-14 in north track. |
| N-14 | 203286 | 19 | 33 | BC | AB | |
| S-22 | 212256 | [4] 22 | 1 | B | ---- | |
| H-29 | 206264 | 20 | 12 | AB | AB | Yellow dog curve; high rail; spiral of 3° 6′ curve with superelevation of 3½ inches; 20,562,000 tons of traffic. |
| H-30 | 206264 | 20 | 10 | AB | AB | |
| H-15 | 212256 | [4] 22 | 2 | B | ---- | |

[1] These numbers correspond to same heats previously studied. (B. S. Tech. Paper No. 363.)
[2] The rails in previous tests (B. S. Tech. Paper No. 363) were taken from "a middle ingot" of the heat.
[3] This nomenclature of AB and BC is the same as was used in the previous report. Because of the relatively large crop the manufacturer marked the top rail the B rail and second rail the C rail. To distinguish these, they were called the AB and BC rails, respectively.
[4] No specimens from rails from this heat, which was a standard heat, were tested before being placed in service. It is comparable to heat 21 of previous report.

TABLE 2.—*Record of rails returned from service on the Canadian Pacific Railway*

| Railroad company No. | Manufacturers heat No. | Bureau of Standards heat No.[1] | Ingot No. | Position in ingot of rail tested | | Date removed from service | Total service tonnage, including locomotive | Service condition as given by railroad |
|---|---|---|---|---|---|---|---|---|
| | | | | After service | Before service[2] | | | |
| 5157-5-A | 5157 | 1 | 5 | A | A | Jan. 19, 1928 | 12, 130, 000 | All rails laid on tangent in north line of rails in eastbound track of double-track Winchester subdivision immediately east of Dorval, Province of Quebec, July 30, 1926. |
| 5157-12-A | 5157 | 1 | 12 | A | A | Jan. 27, 1928 | 12, 330, 000 | |
| 5157-8-B | 5157 | 1 | 8 | B | B | Jan. 19, 1928 | 12, 130, 000 | |
| 5157-13-B | 5157 | 1 | 13 | B | B | Jan. 27, 1928 | 12, 330, 000 | |
| 5159-10-A | 5159 | 3 | 10 | A | A | ----do---- | 12, 330, 000 | |
| 5159-21-A | 5159 | 3 | 21 | A | A | Jan. 19, 1928 | 12, 130, 000 | |
| 5159-1-B | 5159 | 3 | 1 | B | B | Jan. 27, 1928 | 12, 330, 000 | |
| 5159-21-B | 5159 | 3 | 21 | B | B | Jan. 19, 1928 | 12, 130, 000 | |
| 5157-13-D | 5157 | 1 | 13 | D | ---- | ---------- | 17, 030, 000 | |
| 5159-21-E | 5159 | 3 | 21 | E | ---- | ---------- | 17, 305, 000 | |

[1] These heat numbers correspond to same heats previously tested (B. S. Tech. Paper No. 363).
[2] The rails in previous tests were taken from "a middle ingot" of the heat.

It will be noted from Table 1 that the rails returned from the Baltimore & Ohio Railroad were obtained from three heats which had been poured in sink-head ingots, which had been previously tested, and from one standard heat. Unfortunately, it was not possible to locate rails from the standard heat (No. 21) previously tested.

It will be noted from Table 2 that all rails from the two heats returned from service on the Canadian Pacific Railway were rolled from the special heats killed with aluminum and poured in sink-head ingots. Unfortunately, none of the rails from the standard ingot heats could be located.

In the previous work, material was not available to determine the variation in the endurance limits that might be shown between steel from rails from the same position in different ingots from the same

heat. A measure of this difference was desirable in order to be able to differentiate between any change that might be due to overstressing or understressing and the differences inherent between rails from the same position in different ingots.

Accordingly, two A and B rails, respectively, representing each heat were removed from service. This made possible, the service conditions of comparable rails having been the same, a comparison of endurance properties in each instance of the steel from two rails from the same position in two distinct ingots of each heat and also a direct comparison with the steel from similar rail before service. At a later date a D and an E rail from the same ingots as the A and B rails of heats 5157 and 5159, respectively, were obtained from the Canadian Pacific for test as discussed more fully later in the report. For convenience of reference to the earlier work on the new rail, the same designations have been given in the tables as were previously used.

The conditions of service for rail removed from track on the Baltimore & Ohio Railroad were stated by Mr. Stimson to have been as follows:

* * * For your information I quote the following statement, which indicates rail designation, process, heat number, and ingot number:

| | | | |
|---|---|---|---|
| Yellow dog tangent: | | | |
| S–13, Gathmann | 203286 | C | 10 |
| N–14, Gathmann | 203286 | C | 33 |
| S–22, Regular | 212256 | B | 1 |
| Yellow dog curve: | | | |
| H–29, Gathmann | 206264 | B | 12 |
| H–30, Gathmann | 206264 | B | 10 |
| H–15, Regular | 212256 | B | 2 |
| Bridge 323: | | | |
| L–15, Gathmann | 203284 | C | 13 |
| L–17, Gathmann | 203284 | C | 15 |

The designations S, N, H, and L refer to north, south, high, and low. As mentioned in my previous letter. the tonnage which traveled over the first six rails listed was 20,562,000 tons, and over the last two, 21,738,000 tons.

There is, as you will notice, considerable difference in the wear on these rails, but this is easily explained when it is understood that rails from bridge 323 were on the low side of a 9° 25′ curve with a superelevation of 5½ inches, while the other rails were either upon the spiral of a 3° 6′ curve with a superelevation of 3½ inches or upon tangent.

The conditions of service for the rail removed from track on the Canadian Pacific Railway were stated by Mr. Fairbairn to have been as follows:

All of the test pieces sent you were laid on tangent in the north line of rails in the eastbound track of our double-track Winchester subdivision immediately east of Dorval, Province of Quebec, July 30, 1926.

The original pieces sent you were removed from the track on January 19, 1928, after approximately 12,130,000 tons, including locomotives, had passed over them.

About 10 per cent of the tonnage in each case was passenger trains hauled by G–1 and G–3 class locomotives. The remainder was freight tonnage handled chiefly by D–10 class locomotives, with occasional N–3 class locomotive. I am attaching wheel-load diagrams of these locomotives.

The diagram of wheel loads mentioned is given in Figure 1.

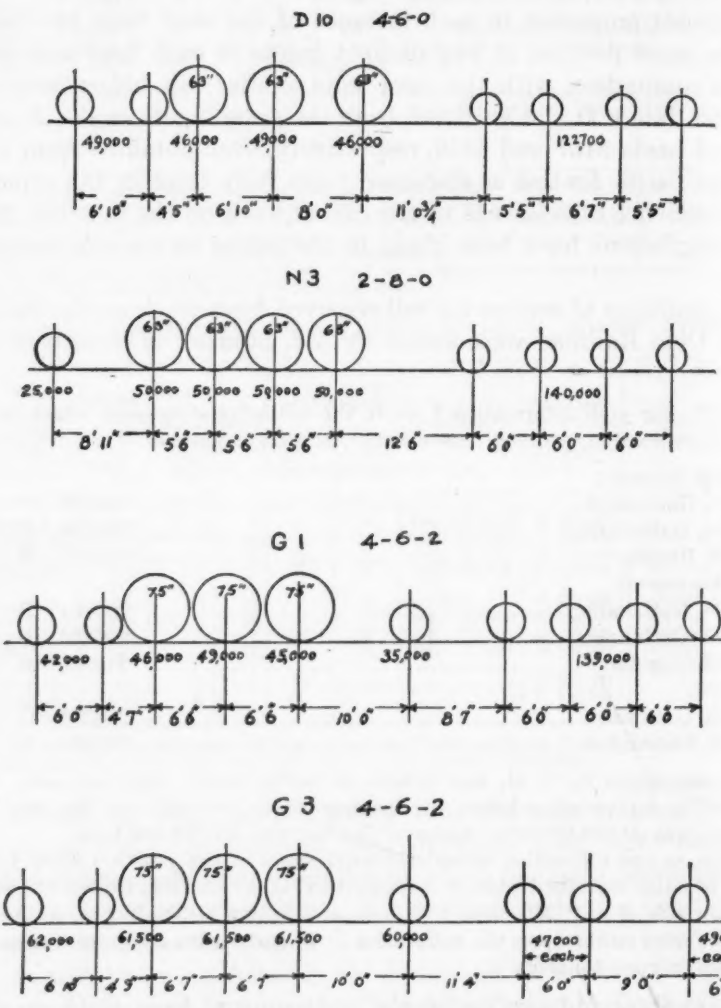


FIGURE 1.—*Wheel load diagram. Canadian Pacific Railway locomotives. Classes D–10, N–3, G–1, and G–3*

## III. TEST PROCEDURE

In order that the test conditions should be as nearly comparable as possible the specimens were taken in the same manner and the same test equipment and test procedure were used as in the previous work.

For convenience, a figure showing location of test specimens is repeated here. (Fig. 2.) Duplicate contiguous (end to end) specimens were taken in each case, as previously, giving 12 specimens at a distance of between 4½ to 6 feet from the end of the rail.

At least one of each series of test specimens was subjected to 18,000,000 to 20,000,000 cycles of stress at a value just under the endurance limit. All other specimens were run at least 10,000,000 cycles if failure did not occur previously and then in some instances

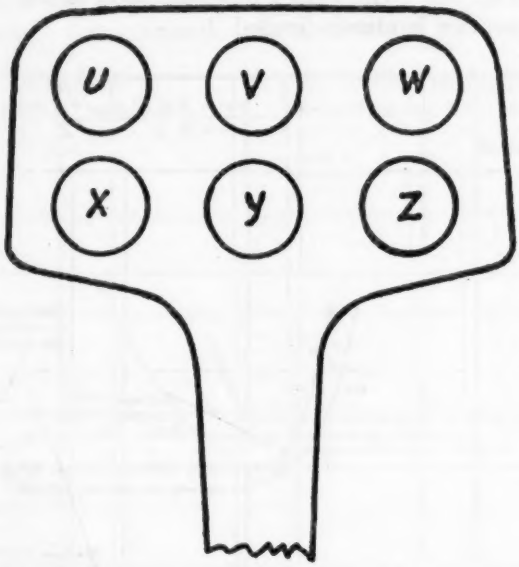


FIGURE 2.—*Location of endurance-test specimens*

tested at a stress appreciably above the endurance limit to determine whether strengthening by understressing had occurred during testing.

## IV. RESULTS OF TESTS OF RAIL AFTER SERVICE ON BALTIMORE & OHIO RAILROAD

### 1. ENDURANCE TESTS OF STEEL FROM RAILS RETURNED FROM BALTIMORE & OHIO RAILROAD

The results of all endurance tests on rail steels returned from service on the Baltimore & Ohio Railroad are given in the customary S–N diagrams in Figures 3 to 7, inclusive.

In order to compare the results shown by specimens from the rails removed from service with the similar data on specimens from new rail from the same heats the endurance curves of the latter have been drawn in on the respective charts.

Tests were not made on specimens from new rails of heat No. 22 (fig. 3), which was a standard heat. A comparison of the effect of service on standard ingot rail, however, is available by comparing the endurance curves of the steel from the B rails of heat 22 with the results of similar tests on steel from the B rails from four similar standard heats (Nos. 8, 9, 10, 21, of previous report) tested before service. The range (maximum and minimum values) and average endurance limits shown by these four rail steels are indicated in the chart. (Fig. 3.) The range of endurance limits of all B rails tested before service is also indicated.

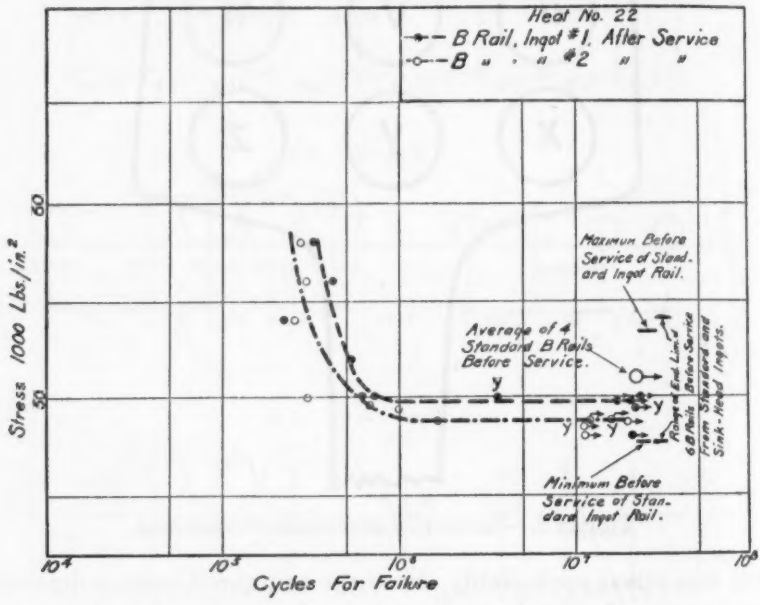


FIGURE 3.—*Endurance curves of steel from B rails from heat No. 22 after service and indicated range of endurance limits of similar B rails before service*

It is evident from the data that the endurance limit of the rail steel after service is well within the limits obtained from similar rail before service.

The conclusion is therefore evident that the service conditions in this instance have had no apparent effect upon the endurance properties of this rail steel.

It is possible in only one instance to make a direct comparison of the endurance limit of steel from a new and serviced rail from the same heat (No. 20) and position (AB) [5] in the ingot. This is shown in Figure 4. The endurance limit of the steel from the AB

[5] The same special nomenclature is used here for these rails as in the previous report.

rail from one of the middle ingots of the heat before service was 53,600 lbs./in.$^2$ The endurance limits shown by specimens from similar rails (AB) from two different ingots of the same heat were found, after service, to be approximately 53,000 and 54,400 lbs./in.$^2$, respectively.

Specimens from each of these rails that had not failed during test after approximately 20,000,000 cycles were tested at successively higher loads. In one case an endurance limit greater than 60,000 lbs./in.$^2$ and in the other greater than 65,000 lbs./in.$^2$ was obtained when tests were discontinued. As a result of this repeated under-

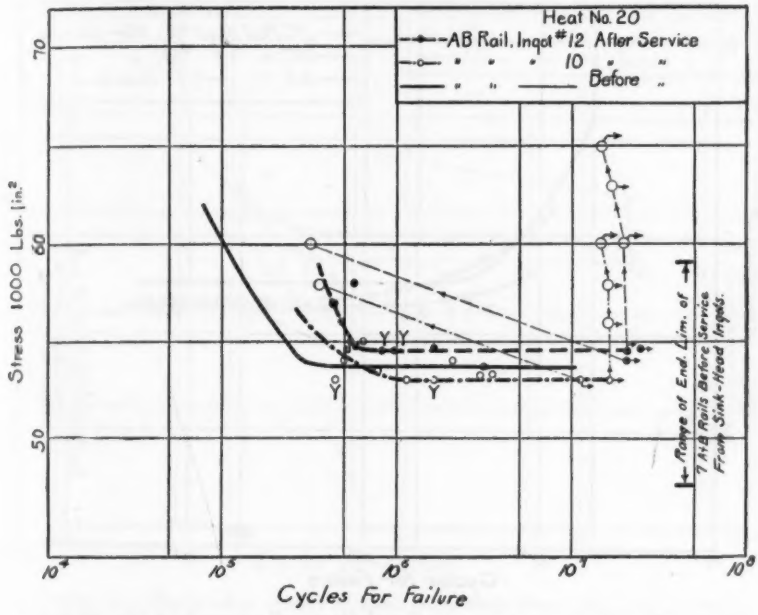


FIGURE 4.—*Endurance curves of steel from AB rails from heat No. 20 before and after service*

stressing, "coaxing," the endurance limits were increased approximately 13 and 19.5 per cent, respectively.

There is therefore no evidence, in this instance also, that the service stresses affected the endurance properties of the steel.

The results of tests on specimens from rails from heats Nos. 18 and 19 are given in Figures 5 and 6. In both instances the rails removed from service were BC rails and are compared with test results from AB rails from the same heat.

The endurance limits shown by the specimens from both of the serviced rails from heat No. 18 (fig. 5) are practically the same at about 56,600 lbs./in.$^2$ and are only slightly less (1,100 lbs./in.$^2$) than

the endurance limit of 57,400 lbs./in.$^2$ shown by the specimens from the AB rail before service.

In the previous work (B. S. Tech. Paper No. 363, p. 292) differences of 2,700 lbs./in.$^2$ were found in the endurance limits shown by specimens from A and B rails from the same ingot. The difference of 1,100 lbs./in.$^2$ noted is therefore probably a difference originally present in the steel. It is also to be noted, as indicated in the figure, that the endurance limits of the steel of these serviced rails is considerably higher than the minimum value shown by similar A and B rail steels before service.

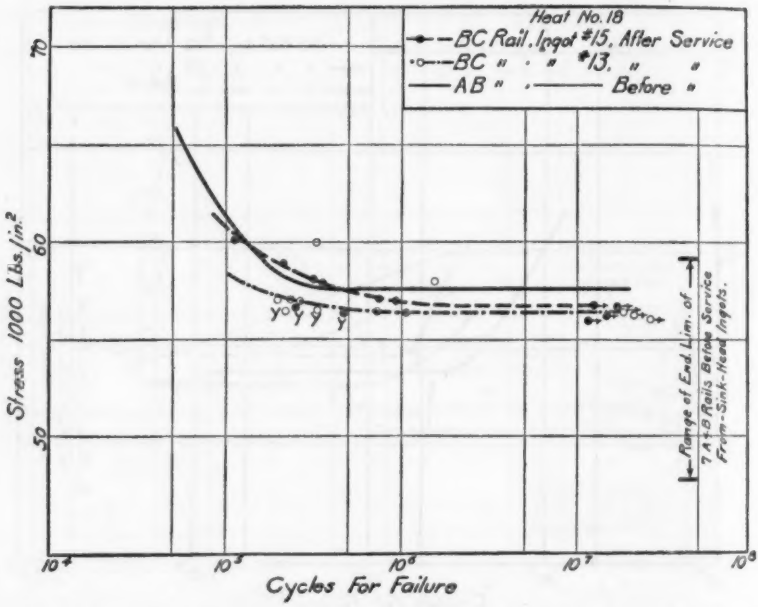


FIGURE 5.—*Endurance curves of steel from rails from heat No. 18 before and after service*

Evidence is again lacking that the service conditions have in any way affected the endurance properties.

The difference in endurance limits that may exist between rails from the same position in different ingots is shown by the data on the serviced rails of heat No. 19. The endurance limit of the steel from one of these BC rails (ingot No. 10) was found to be about 57,600 lbs./in.$^2$ and from the other (ingot No. 33) about 53,500, or 4,100 lbs./in.$^2$ lower.

Both rails were subjected to practically the same traffic conditions. It seems, therefore, a fair assumption that the difference noted was present in the steel before being placed in service.

In both instances the endurance limits are lower than the value shown by specimens from an AB rail from the same heat before serv-

ice. This particular series, however, showed the highest endurance limit of all of the specimens from all rails tested before service, and the value of 53,500 lbs./in.$^2$ shown by the BC rail from ingot No. 33 is practically the same as that obtained on similar tests of an AB rail from heat No. 20 before service. Further, as indicated in the figure, the endurance limits in both instances are well within the limits shown by specimens from similar new rail.

In this case, also, specimens were restressed as indicated. In each instance strengthening by understressing of the test specimen had

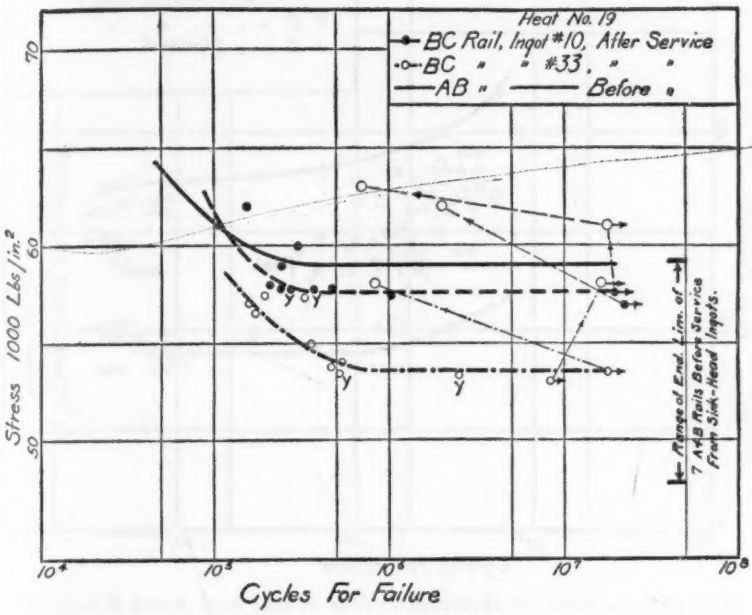


FIGURE 6.—*Endurance curves of steel from rails from heat No. 19 before and after service*

occurred, as shown by the increased endurance limits of approximately 8.5 and 6.0 per cent.

The conclusion is again justified that the service conditions have not affected the endurance properties of the steel.

The results of all of the above-mentioned endurance tests on rail steels returned from service have been plotted in Figure 7. The maximum and minimum values of steel from new rails from the similar heats have also been indicated.

It is apparent that the range of endurance limits shown by the steel from serviced rail is within that of the new rail.

Attention is called to the really random selection of the test specimens representing the cross section of the heads of several rails. If localized defective areas (transverse-fissure nuclei) were present

originally or as a result of service, it would seem probable such areas would have been present in one or more of the test specimens and have caused premature failure.

The grouping of all the test results within the well-defined limits indicated in Figure 7 is strong evidence that such defective areas were not present in the rails tested.

It is also noteworthy that the endurance of steel from the standard rails tested in this instance is lower than that from sink-head rails.

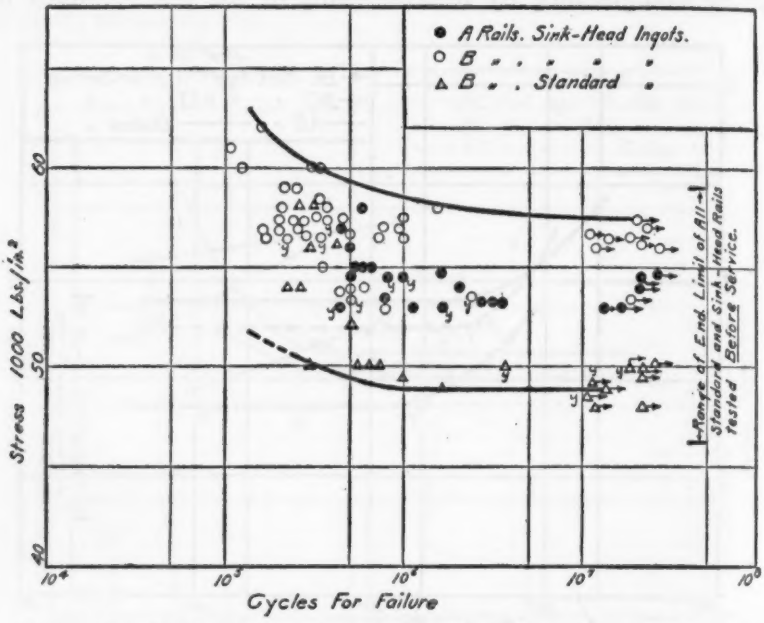


FIGURE 7.—*Results of all endurance tests on steel from A and B rails after service on Baltimore & Ohio Railroad*

## 2. EFFECT OF SURFACE FLOW ON HARDNESS

Some of the rails returned from service on the Baltimore & Ohio Railroad showed appreciable surface flow of the head of the rail. This is indicated in Figure 8, which is a photograph of a section of rail No. L–15 (Table 1), taken about 10 feet from the end.

Rockwell B hardness tests were made at the points indicated to determine the increased hardness due to cold flow.

The average hardness across the upper edge of the section about one-sixteenth inch from top of rail was found to be 106.7. The average hardness along a line about one-fourth inch below the top of the head of the rail was found to be 104.6. The average hardness of the interior of head of the section, as shown by measurements at points indicated, which were taken in the same manner as in the previous report (p. 340, fig. 30), was found to be 100.2.

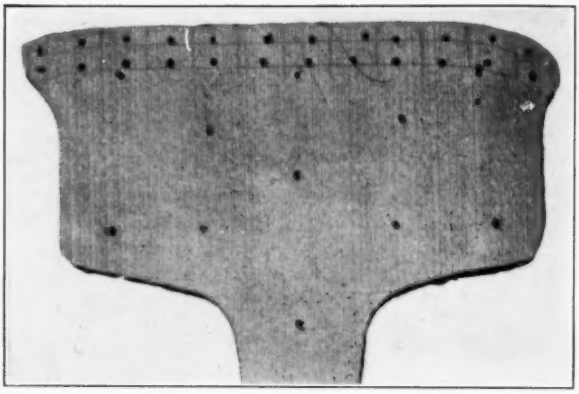

FIGURE 8.—*Section of head of rail L–15 showing surface flow and location of Rockwell B hardness tests*

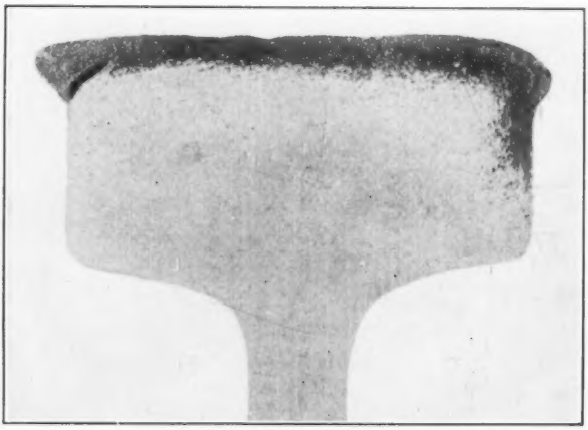

FIGURE 10.—*Section of rail L–15 adjacent to location of endurance test specimens*

Deep etched in hot concentrated HCl for one-half hour. Surface slightly reground. Dark area indicates region of surface flow.



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Tests made along the top edge of new rail from the same heat in the same manner as on the serviced rail indicated no difference in hardness as compared with the interior of the head.

It is evident, therefore, that the surface flow caused an appreciable hardening of a relatively thin surface layer due to the cold working of the metal and that the hardness of the interior of the head was not affected by service.

### 3. EFFECT OF SURFACE HARDENING ON ENDURANCE PROPERTIES

In view of the surface flow and resultant increase in hardness of the top of this rail, which is undoubtedly a common occurrence in service, it was considered desirable to determine what effect, if any, this surface hardening might have on the endurance properties of the steel adjacent to the cold-worked area.

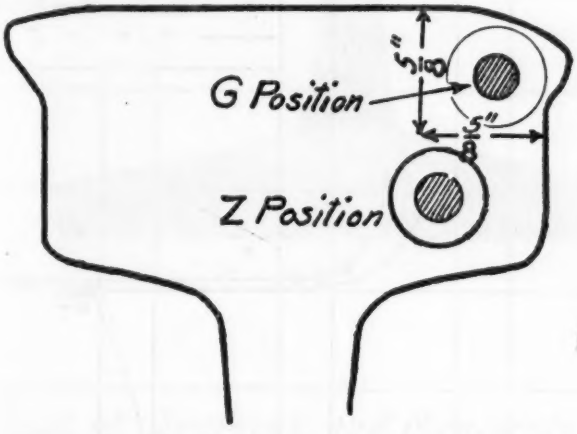



FIGURE 9.—*Location of endurance-test specimens in rail L–15*

Shaded portions show approximate location of test area

Nine endurance-test specimens were taken, therefore, contiguously along the length of the rail on the same side from the positions marked G and Z in Figure 9. The Z position is identical with the Z position of tests on other rails. (Fig. 2.) The G position, however, was taken as near the gage corner as it was possible to machine out the test specimens, which was limited by the one-half inch diameter necessary for the tapered end of the specimen. The location of the test area corresponded approximately to the shaded area in Figure 9.

Rockwell B hardness tests on the heads of the broken specimens after test at points corresponding to the periphery of the minimum sections gave a maximum value greater than 100 (102.5) in only one instance, which indicates that the test areas of the several specimens were not in the work-hardened layer, but just beneath it.

The relation of location of test specimens to depth of cold working of the head is evident by comparing with Figure 10, which is a photograph of a section of the same rail deep etched in hot concentrated HCl, the surface of which was ground off slightly after deep etching. The cold-worked layer being more deeply attacked by the acid was not removed in grinding, and is therefore outlined approximately by the dark area at top of section.

The results of these endurance tests are given in Figure 11.

It is evident that the endurance limits of specimens from the G and Z positions are practically identical.

It may be concluded that the degree of cold flow of surface present in this instance has not affected the endurance properties of the

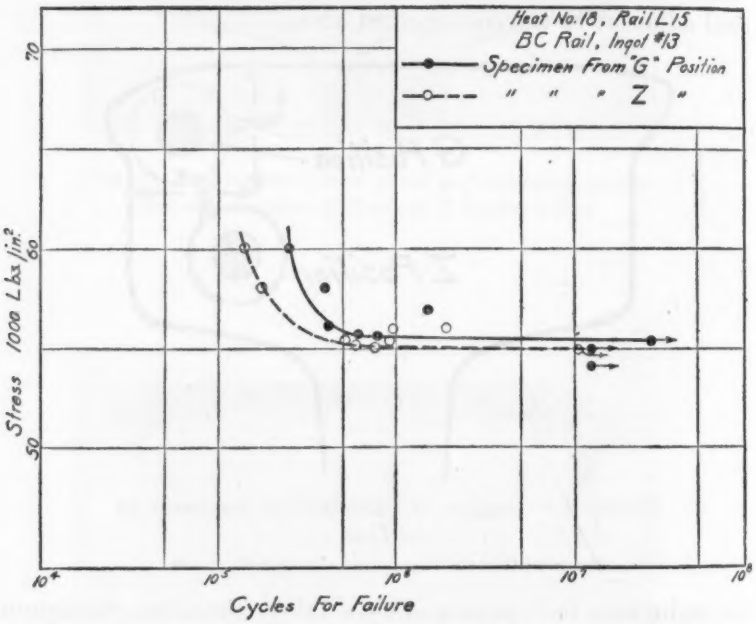
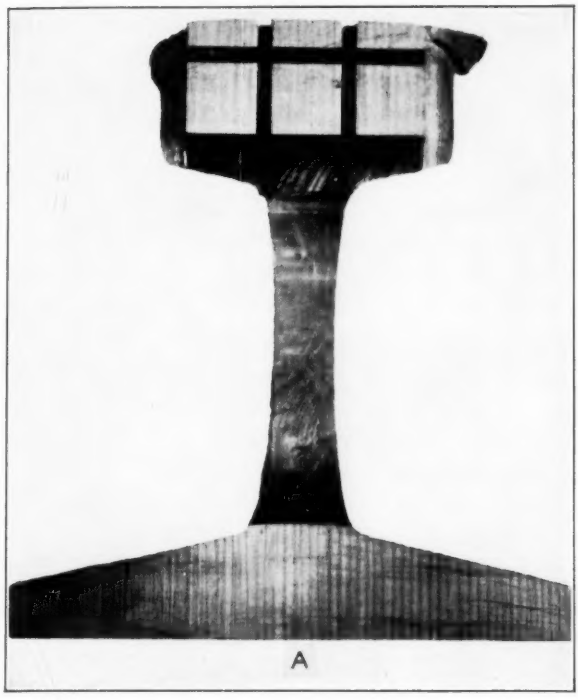



FIGURE 11.—*Effect of surface flow on endurance of steel in head of rail L–15*

Specimens taken as indicated in Figure 9

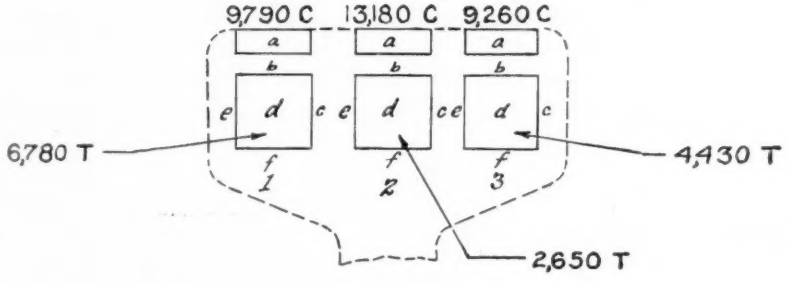
steel along the adjacent section from which the G test specimens were taken.

It should be emphasized, however, that it is the endurance properties of the steel per se that are not affected, since machining out the test specimens will in large measure relieve internal stresses that may have been in the rail due to distortion by cold flow. Such internal stresses are probably not present, at least to the same magnitude, in the test specimens. It is known that the endurance limit of steel under reversed axial stresses is affected by the presence of initial stresses in tension or compression. The resistance of the rail,



A

Sketch of Surfaces and Measuring Points.




B

FIGURE 12

*A*, Method of sectioning rail for internal stress measurements
*B*, Distribution of internal stress in head of rail after service. lbs/in.$^2$ *c*, compression; *t*, tension



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therefore, to the fatigue stresses imposed in track might be different from the values indicated in Figure 11, depending upon the magnitude and relative direction of the internal stresses and the service stresses.

The degree of internal stress that may have been present in some of the rails is indicated by the results of tests described below under internal stress.

The data do show, however, that such maximum stresses as may have been developed in service in this instance did not cause incipient internal failure (shattering) of the steel. Had such occurred, a marked "scatter" of the results of the fatigue test would have occurred.

This is of special significance in view of the fact that the internal nuclei from which transverse fissures progress are found predominantly on the gage side of the rail.

It is of interest to note that the endurance limits shown in this case by a curve obtained from specimens taken contiguously along the length of the rail is in agreement with the results (fig. 5) obtained on tests of the same rail in which the specimens were taken across the section of the rail (fig. 2).

### 4. INTERNAL STRESS IN RAIL AFTER SERVICE

Rail No. L–17 showed the greatest amount of surface flow of the head. It was, therefore, used for the determination of the degree of internal stress that might be present in these rails after removal from service.

The ends of a section approximately 10 inches long were divided into six parts by cutting away about one-quarter inch as shown in Figure 12 (*a*). The raised surfaces were then ground and lapped flat and parallel and positions identified as shown in Figure 12 (*b*).

The distances between corresponding points on the end surfaces were then measured in the gage section of the Bureau of Standards.

The six parallel sections were then machined out of the head of the rail. The distances between corresponding points were then redetermined.

The results of the determinations are given in Table 3.

It is evident that the top surface of the rail as represented by the *a* sections was in compression in a longitudinal direction and the central portion of the railhead as represented by the *d* sections was in tension

On the assumption that the modulus of elasticity of the steel is 30,000,000 lbs./in.$^2$ and that no bending occurred in the test sections, the internal stresses present in pounds per square inch in the rail were as given in Figure 12 (*b*) at the points indicated.

TABLE 3.—*Dimensional changes in cutting head of rail for determination of internal stress*

| Position | Length | | |
|---|---|---|---|
| | Before cutting | After cutting | Change |
| | *Inches* | *Inches* | *Inch* |
| 1. *a* | 9.74552 | 9.74870 | +0.00318 |
| *b* | 9.74549 | 9.74365 | −.00184 |
| *c* | 9.74545 | 9.74357 | −.00088 |
| *d* | 9.74545 | 9.74305 | −.00240 |
| *e* | 9.74546 | 9.74239 | −.00307 |
| *f* | 9.74543 | 9.74243 | −.00300 |
| Average of changes *b*, *c*, *e*, and *f* | | | −.00220 |
| 2. *a* | 9.74553 | 9.74981 | +.00428 |
| *b* | 9.74549 | 9.74534 | −.00015 |
| *c* | 9.74547 | 9.74482 | −.00065 |
| *d* | 9.74546 | 9.74458 | −.00088 |
| *e* | 9.74546 | 9.74426 | −.00120 |
| *f* | 9.74545 | 9.74402 | −.00143 |
| Average of changes *b*, *c*, *e*, and *f* | | | −.00086 |
| 3. *a* | 9.74555 | 9.74856 | +.00301 |
| *b* | 9.74553 | 9.74397 | −.00156 |
| *c* | 9.74548 | 9.74388 | −.00160 |
| *d* | 9.74549 | 9.74411 | −.00138 |
| *e* | 9.74548 | 9.74412 | −.00136 |
| *f* | 9.74546 | 9.74422 | −.00124 |
| Average of changes *b*, *c*, *e*, and *f* | | | −.00144 |

The fact that a change in length occurred in the sections machined out of the rail confirms a previous statement that the endurance limits, as determined on individual specimens removed from the rail, represent the properties of the steel probably relatively free from internal stress.

The internal stresses would, of course, add to or decrease the resultant maximum stresses imposed by service according to their direction, an internal tensile stress increasing the maximum resultant stress imposed in tension but correspondingly decreasing the resultant maximum stress in compression. Internal stresses merely produce a shift in the total range of stress in the rail.

## 5. RELATION OF ENDURANCE TO TENSILE PROPERTIES

Tensile tests were made using standard 0.505-inch diameter test specimens with 2-inch gage length taken from the O position. The results, together with respective endurance limits and endurance ratios, are given in Table 4.

A comparison of the endurance ratios of serviced rails with those of new rail, as given in Tables 1 and 2, page 283, of previous report, shows that they have approximately the same average value of 0.41.

TABLE 4.—*Tensile and endurance properties of rail steels after service, Baltimore & Ohio Railroad*

| Railroad company marking | Bureau of Standards heat No. | Elongation, per cent in 2 inches | Reduction of area | Yield point | Tensile strength | Endurance limit | Endurance ratio |
|---|---|---|---|---|---|---|---|
| | | | *Per cent* | *Lbs./in.²* | *Lbs./in.²* | *Lbs./in.²* | |
| L-15 | 18 | 13 | 7.5 | 66,800 | 129,500 | 56,500 | 0.44 |
| L-17 | 18 | 13 | 8.0 | 74,800 | 129,200 | 56,750 | .44 |
| S-13 | 19 | 13 | 8.0 | 76,200 | 128,800 | 57,000 | .44 |
| N-14 | 19 | 13 | 8.0 | 74,700 | 129,000 | 53,500 | .41 |
| S-22 | 22 | 13 | 8.0 | 71,200 | 126,700 | 49,500 | .39 |
| H-29 | 20 | 13 | 8.0 | 76,600 | 129,000 | 54,500 | .42 |
| H-30 | 20 | 13 | 8.0 | 70,200 | 128,900 | 53,000 | .41 |
| H-15 | 22 | 13 | 8.0 | 77,500 | 128,200 | 48,750 | .38 |
| Average | | | | | | | .41 |

## 6. SUMMARY OF RESULTS OF TESTS ON BALTIMORE & OHIO RAILS

A series of endurance tests have been made on specimens taken from eight A and B rails from four distinct heats which had been subjected to over 20,500,000 tons of traffic in track of the Baltimore & Ohio Railroad. Six of the rails were A and B rails from three special heats killed with aluminum and poured in sink-head ingots, from which similar tests had been made on new rail previous to service. The two other rails were B rails from a normal heat cast in standard molds.

Direct comparison of the results of the endurance tests on the steel from the serviced rails from the sink-head ingots with tests of the steel from new rail from the same heats was, therefore, possible. Comparison of the results of tests from the serviced rail from the standard ingots could be made with the results of similar tests on specimens from similar new rails from several standard heats.

The test rails were also chosen so that it was possible to determine the variation in endurance properties of specimens from rails from corresponding portions, A and B rails, of ingots from the same heat. Such comparison before service was not made.

The maximum difference noted in endurance limit between the rail steels from comparable positions in the ingots from the same heat after service was 3,500 lbs./in.² for the B rails from heat No. 19. In the three other instances, where comparison was possible, the differences were found to be 1,500, 750, and 250 lbs./in.², respectively.

The maximum difference found in the endurance limit between steels from serviced and unserviced rails from the same heat was also in heat No. 19, in which steel from the A rail before service had an endurance limit of 59,200 lbs./in.² and a B rail after service 53,500 lbs./in.², a difference of 5,700 lbs./in.² In the other instances the differences were much less than this, being less than approximately 2,000 lbs./in.² in each case.

The results of all endurance tests of serviced rails (fig. 7) indicated that the service conditions imposed caused no apparent change in the endurance properties of the steel; apparently no "overstressing" or "understressing" had occurred.

Tests were also made which showed that appreciable surface flow and hardening of the top of the rail in service did not affect the endurance properties of the steel in the head of the rail immediately below the work-hardened layer.

It is therefore indicated that the service stresses in the head of the rails were not greater than the endurance limit of the steel.

## V. RESULTS OF TESTS OF RAILS AFTER SERVICE ON CANADIAN PACIFIC RAILWAY

### 1. ENDURANCE TESTS OF STEEL FROM RAILS FROM CANADIAN PACIFIC RAILWAY

The results of all endurance tests on rail steels returned from service on the Canadian Pacific Railway are given in Figures 13 to

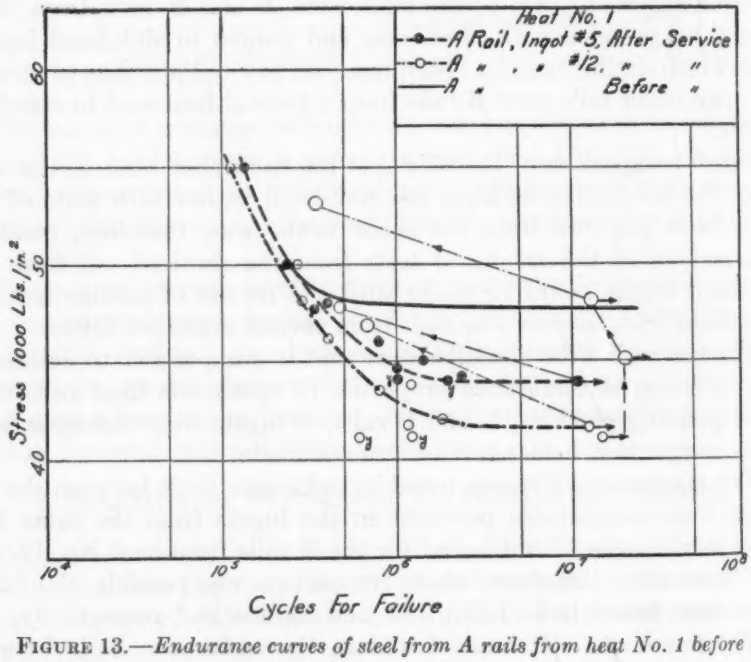


FIGURE 13.—*Endurance curves of steel from A rails from heat No. 1 before and after service*

16 plotted in the same manner as the similar data on serviced rails from the Baltimore & Ohio Railroad. Comparison of the results with tests of new rails before service is also made in the same manner.

In each case it is possible to make a direct comparison of the endurance properties of the steel from rails from similar positions (A and B rails) in ingots from the same heats both before and after service.

The results of the tests of specimens from the two A rails from heat No. 1 after service are given in Figure 13, together with the results from the A rail from a middle ingot of the same heat before service.

The endurance limits of 44,000 and 41,500 lbs./in.$^2$ shown by the serviced material are lower than the value of 47,800 lbs./in.$^2$ shown by the steel from the new rail from the same heat.

The difference in value between the two serviced rails of 2,500 lbs./in.$^2$ indicates the variation that may occur between rails from similar positions in different ingots within the heat, the service conditions having been the same. This is less than the maximum difference of 4,100 lbs./in.$^2$ noted (BC rail, heat 19) in similar tests of the rail from the Baltimore & Ohio. (Fig. 6.)

The lowest value obtained on similar tests of rails before service from all heats was 46,200 lbs./in.$^2$ except for the value of 41,000 lbs./in.$^2$ indicated by specimens taken contiguously (instead of from adjacent positions (fig. 2) from the U position of the B rail of heat 11 in the previous report (fig. 21, p. 293).

The data for the curve for the A rail from ingot No. 5 are relatively uniform and characteristic of homogeneous material. The data for the A rail from ingot No. 12 are also quite uniform except for the specimens from the Y position which indicate some inhomogeneity.

As was pointed out in the previous report on new rail (pp. 277, 289), this lower endurance value of the Y specimen might be expected and was found in new rails.

The cause for the lower values is not evident. The magnitude of the difference is not great. In view of the relatively small number of tests on new rail by which the range of endurance limits to be expected was determined, the fact that tests were made on only one A rail from this heat before service and the slightly lower value shown by the new B rail from heat 11 all indicate that equally lower values might have been found in new A rail from this same heat had tests been made.

It is, of course, possible that the lower values may be the result of "overstressing" in service. In view of the fact, however, that the specimens that did not fail could be restressed, as indicated in the figure, at values appreciably above their primary endurance limit proves that they were "understressed" as the result of testing and tends to support the above indications that the steel was not in an overstressed condition as removed from track previous to testing.

The results of tests on the specimens from the A rails from heat No. 3 are given in Figure 14.

The data on the steel from the rail from ingot No. 10 indicate an endurance limit of 52,000 lbs./in.$^2$, 3,000 lbs./in.$^2$ lower than the value indicated by tests of specimens from a new A rail from the same heat.

The individual tests showed some "scatter" indicative of inhomogeneity in the steel.

The test data for the steel from the A rail from ingot No. 21 of this heat showed a very marked "scatter," indicating a corresponding marked inhomogeneity of the steel. An endurance curve has been drawn through the plotted points showing two specimens that had not failed at approximately 44,000 lbs./in.$^2$ stress after more than 11,100,000 cycles of stress.

One of these specimens (Y) was restressed at 50,000 lbs./in.$^2$ and did not fail after an additional 11,000,000 cycles of stress. The

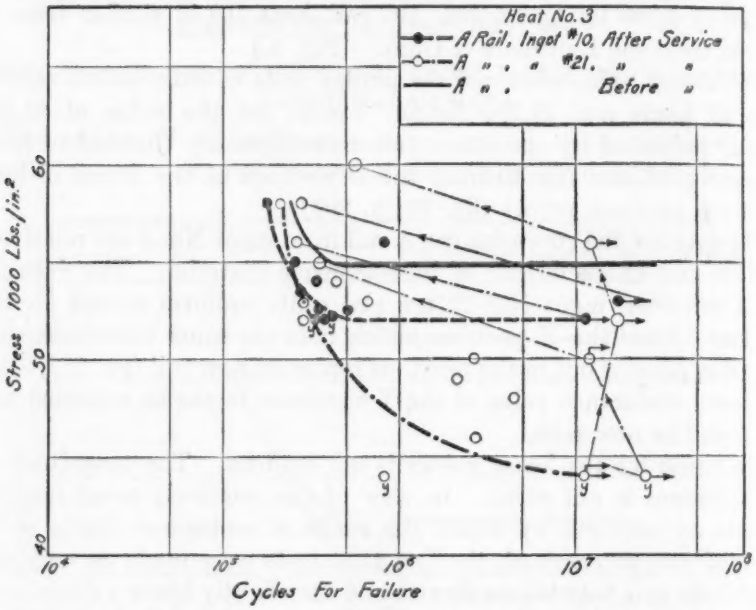


FIGURE 14.—*Endurance curves of steel from A rails from heat No. 3 before and after service*

stress was then increased to 55,000 lbs./in.$^2$, and the specimen had an endurance life equal to that shown by specimens from the new rail of this heat.

The other specimen, which was from a Z position (fig. 2), was restressed as indicated and finally failed at 60,000 lbs./in.$^2$

A probable cause for the marked scatter of results was the presence of internal cracks in the rail when removed from track, as discussed later in this report.

The results of tests on the steel from the B rails from heat No. 1 are given in Figure 15. In both instances the "scatter" is quite marked. In fact, no endurance limit was determinable from the 12 test specimens from the rail from ingot No. 13, all specimens

failing. The general trend of the data is indicated by the curve. It is noteworthy that failures occurred in several instances after a relatively large number of reversals of stress—in one instance after 9,000,000 cycles at 40,000 lbs./in.$^2$

The wide "scatter" in this instance was also found to be due to the presence of internal cracks in the rail when removed from service.

In view of the wide scatter of results obtained, an additional group of six specimens was taken from another section of the same rail and tested independently. The results of these additional tests

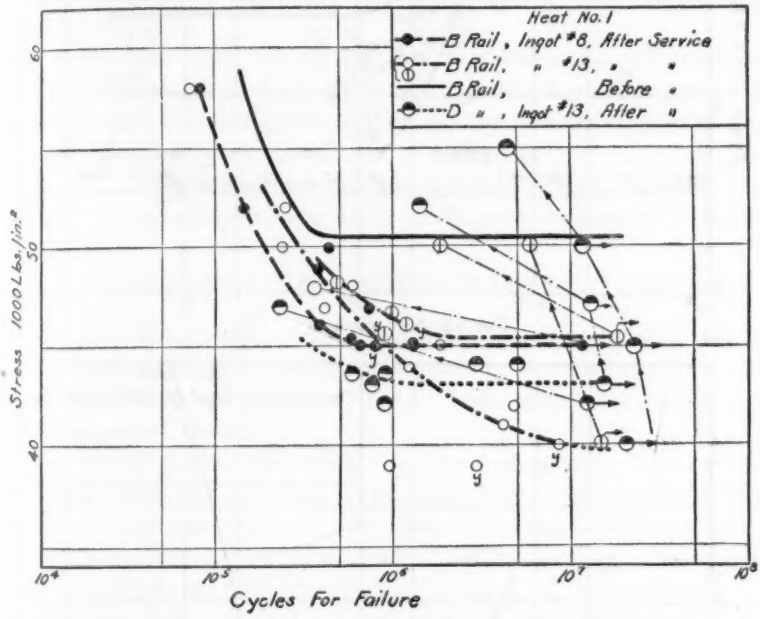


FIGURE 15.—*Endurance curves of steel from B and D rails from heat No. 1 before and after service*

are also included in Figure 15, but distinguished from the first group by the vertical line through the open circle used to indicate the test results of the first group.

The results of the further tests show that the steel at the second location has an endurance limit of approximately 45,000 lbs./in.$^2$ and emphasizes the fact that the wide scatter shown by the first group was due to inhomogeneities (cracks) in the steel.

The endurance limit of 45,000 lbs./in.$^2$ indicated by the tests from the rail from ingot No. 8 is lower than that of the comparison rail but greater than the values indicated by the tests of the serviced A rails from the same heat.

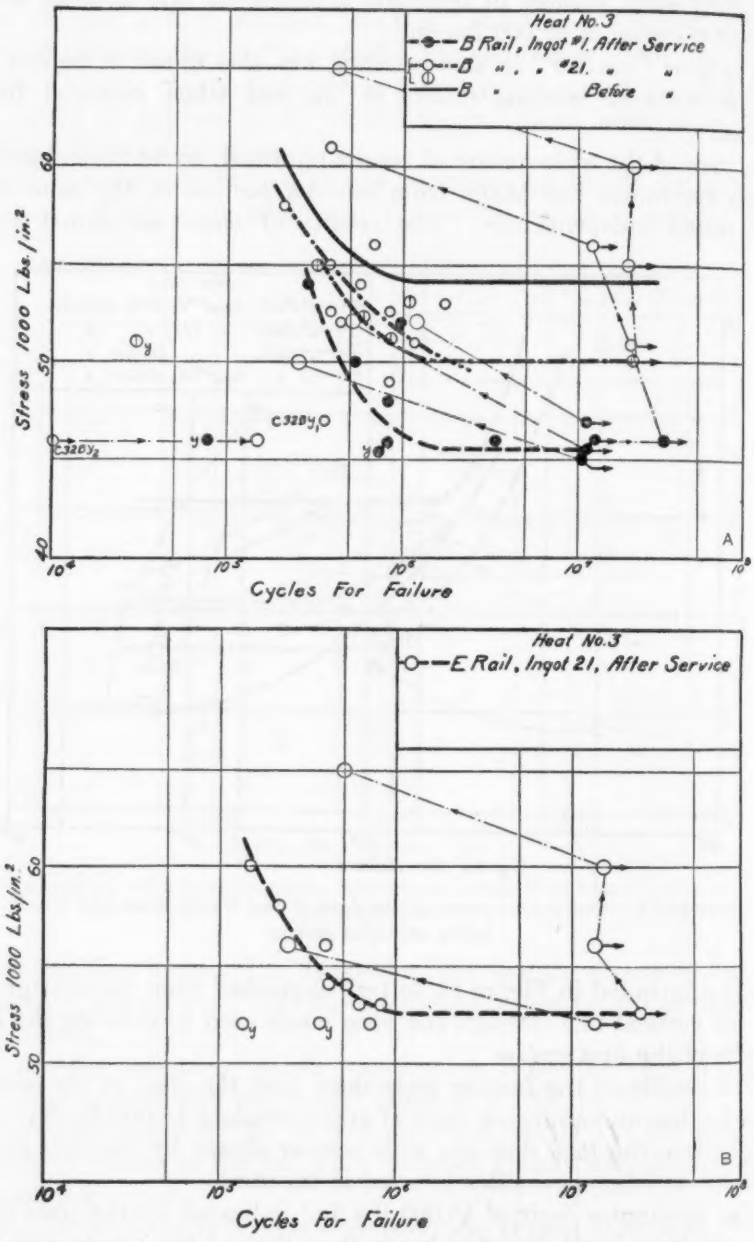


FIGURE 16.—*a* Endurance curves of steel from B rails from heat No. 3 before and after service; *b* endurance curve of steel from E rail from ingot No. 21, heat No. 3

The results of tests of specimens from the B rails from heat No. 3 are given in Figure 16 (*a*). The results in this series are similar to those for the B rails from heat No. 1 after service. (Fig. 15.) There is a marked "scatter" in both instances but particularly for the B rail from ingot No. 21, for which no endurance limit was determinable due to the scatter of results. The approximate trend of the endurance curve is indicated. The cause of this "scatter" was again found to be the presence of internal cracks, as discussed later.

Tests were made at a later date on six additional specimens taken from another section of the same rail. These, except for the Y specimen, gave quite consistent results, indicating an endurance limit of 50,000 lbs./in.$^2$

The endurance limit shown by the tests from the rail from ingot No. 1 at 46,400 lbs./in.$^2$ is appreciably lower than the comparison rail steel tested before service but is slightly greater than was shown by one of the B rails from heat No. 1. (Fig. 15.)

Most of the specimens that did not fail during tests were restressed at values appreciably above their indicated respective endurance limits.

### (a) CAUSE OF "SCATTER" OBSERVED IN ENDURANCE TESTS

Moore and Kommers [6] state "Marked scatter of fatigue-test data for a metal may be interpreted as indicating nonuniformity of structure of the metal, dirty metal, metal with minute cracks in it, or badly segregated metal."

It was thought that the wide "scatter" shown by the test specimens from several of the rails from heats Nos. 1 and 3 might be due to internal cracks in the rail. A careful examination was, therefore, made of some of the test specimens before testing. In specimen C32BY–2, which was from the Y position in the B rail from ingot No. 21 of heat No. 3 (fig. 16 (*a*)), a crack was apparent at a magnification of about 50 diameters on the surface of the specimen.

This specimen was placed in the fatigue-testing machine and run 10,000 cycles at 46,000 lbs./in.$^2$ The computed stress at the location of the crack was approximately 31,700 lbs./in.$^2$ Reexamination of the specimen showed that the crack had widened and extended in length to about 3½ mm around the circumference of the test specimen. The appearance of a portion of the crack after stressing is shown in Figure 17.

The specimen was replaced in the testing machine at the same load as previously, 46,000 lbs./in.$^2$, and failed at the location of the crack in a total of 155,000 cycles, as indicated in Figure 16.

It is obvious that in machining test specimens from the rails it is entirely a question of chance whether any cracks that might be

[6] Moore, H. F., and Kommers, J. B., The Fatigue of Metals, p. 168; 1927.

present would be located at the test section and at the surface or in the interior of the specimen where they would not be detected by visual examination of the polished surface before test.

This was confirmed by examination of specimen No. C32BY1, which failed prematurely, as is evident in Figure 16 (*a*), after 340,000 cycles at 47,000 lbs./in.$^2$

One of the broken ends was cut longitudinally, polished, and examined microscopically. Several small cracks were found at positions indicated in Figure 18.

The crack at position A appeared appreciably larger than the others. It was approximately one-fourth inch from the surface of specimen and one-fourth inch from the fracture. No evidence of its presence could be found on the surface of the specimen

A portion of this crack is shown in the photograph in Figure 19 (*a*). Another portion is shown in Figure 19 (*b*), obtained after appreciable regrinding and polishing. After etching it was apparent that the crack was principally transcrystalline in character, as is typical of

FIGURE 18.—*Location of cracks found in specimen C32BY1 after test*

fatigue failures. One portion, however, was apparently intercrystalline, as is evident in Figure 20, which shows the portion of crack included in circle in Figure 19 (*b*).

The significance of the intercrystalline character of a portion of the crack is discussed later in the report.

The broken ends of several other specimens which failed apparently prematurely were deep etched in hot hydrochloric acid. In every case transverse cracks were found on the surface of the specimen.

It may be concluded that the wide "scatter" of the test results in this and other rails was due to the presence of minute internal cracks in the rail when removed from service, these cracks causing the premature failure of the test specimens.

In order to confirm this conclusion some of the sections of the heads of several rails were cut as shown in Figure 21. The inner surfaces were polished and deep etched in hot concentrated hydrochloric acid and examined for cracks.

There is given in Figure 22 a photograph showing evidence of the internal cracks in the head of the B rail from ingot No. 21, heat No. 3. These were located in the mid section, somewhat above the

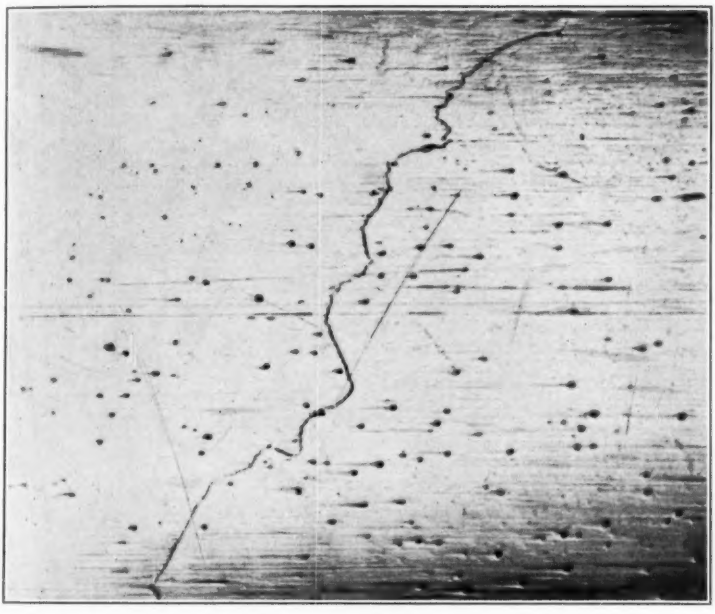

FIGURE 17.—*Portion of crack on surface of fatigue-test specimen from Y position in B rail from ingot No. 21 of heat No. 3 after 10,000 cycles of stress.* × *100*

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FIGURE 19

*A*, Micrograph of portion of crack at position *A* (fig. 18) unetched. × 250
*B*, Another portion after regrinding and polishing

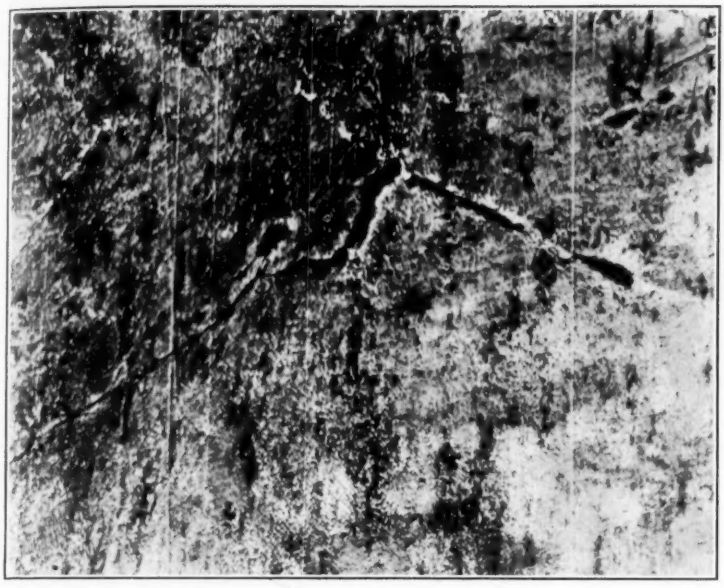

FIGURE 20.—*Micrograph of portion of crack included in circle in Figure 19, B, showing intercrystalline nature, 5 per cent picric acid.* × *500*

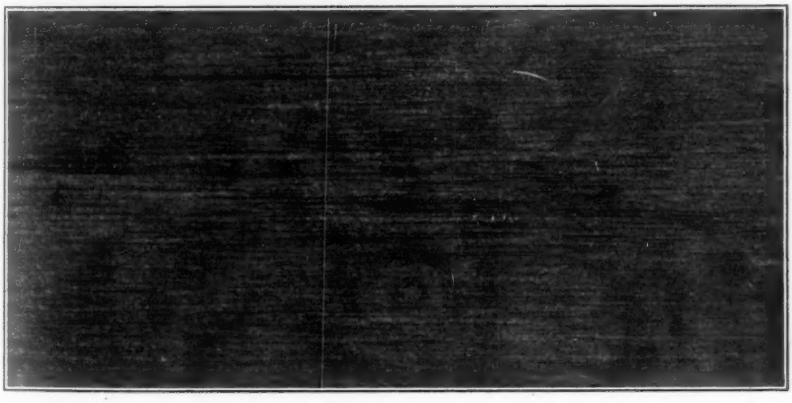

FIGURE 22.—*Longitudinal section of head of B rail from ingot No. 21 from heat No. 3*

Etched hot concentrated HCl 20 minutes. × 1

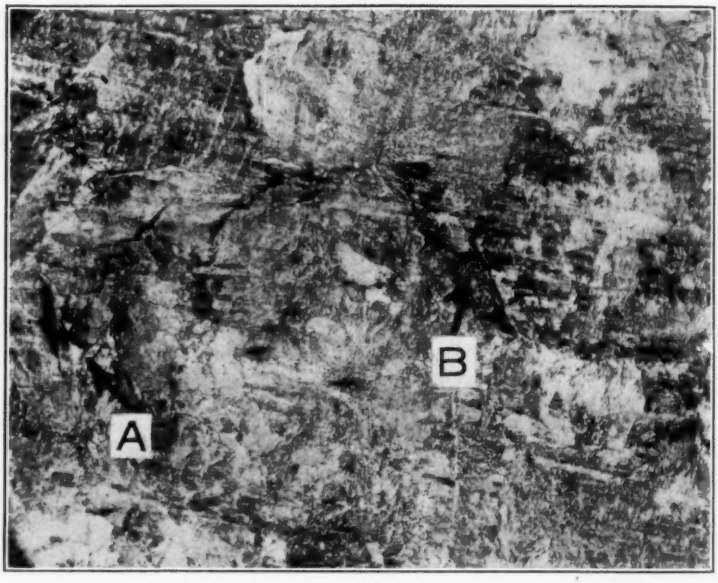


FIGURE 23.—*Portion of a shatter crack found in B rail from ingot No. 1 of heat No. 3*

Etched 2 per cent $HNO_3$. × 100

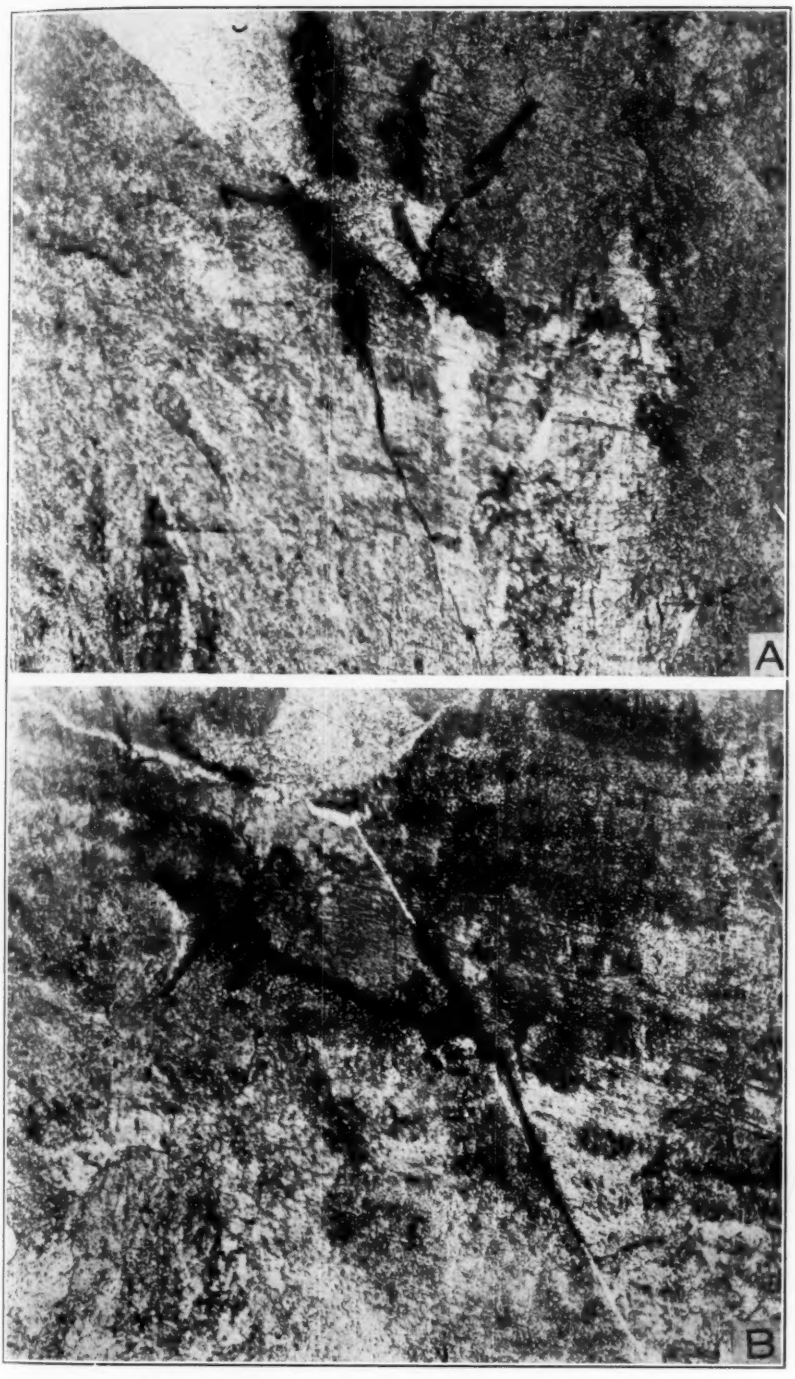


FIGURE 24.—*Portions of cracks shown in Figure 23*

*A*, Portion of crack indicated by arrow "*A*." × 500
*B*, Portion of crack indicated by arrow "B." × 500

FIGURE 25.—*Crack in fatigue-test specimen from a new B rail, heat No. 3*

Etched in hot concentrated HCl. × 25

junction of the head and the web, corresponding approximately to the Y position. (Fig. 2.) Similar cracks were found in a section from the B rail of ingot No. 1 from the same heat.

Detailed examination was made of some of these small transverse cracks. They were principally transcrystalline in nature, but it is believed significant, however, that at the points indicated by the arrows in Figure 23 and shown at 500 diameters in Figure 24 (*a*) and (*b*) that the crack has decidedly intercrystalline characteristics similar to that shown in Figure 20.

It is evident that, had a fatigue-test specimen been taken from the head of the rail in such a manner that this crack or any portion of it

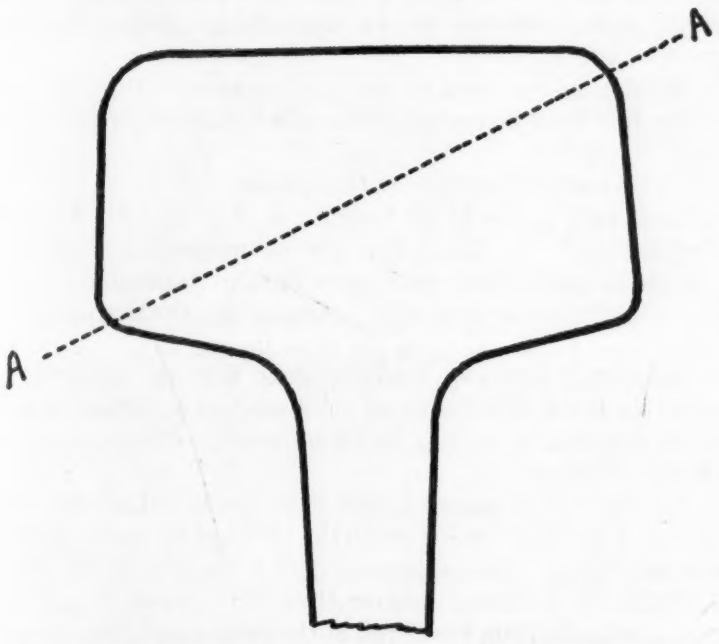


FIGURE 21.—*Method of sectioning head of rail to determine presence of internal cracks. Cut along line AA*

had been located in the test area of the specimen, it would undoubtedly have caused premature failure, as has been pointed out above.

A similar examination was made of a section of the head of the rail adjacent to that from which the fatigue-test specimens for the B rail from ingot No. 8, heat No. 1, were taken. No evidence of internal cracks was found. This would indicate, as might be expected, that internal cracks may be more or less localized in certain sections along the rail and explains the wide scatter of fatigue-test results obtained from one section of a rail while similar tests from another section are relatively uniform.

In the results of the nick and break tests given in Table 12, page 328, of the previous report, it is of interest to note that a large number of the A rails showed evidence of pipe or segregation due to the fact that the hot tops used in pouring the ingots of these two heats were too small, as is discussed fully in that report.

It was thought that the presence of internal cracks found in some of the serviced rails from these two heats might be related to this fact. Accordingly, through the cooperation of Mr. Fairbairn, chief engineer of the Canadian Pacific, the D and the E rails from the same ingots of heats Nos. 1 and 3, respectively, were removed from track and sent in to the bureau for test.

These rails had been in track a longer time and, of course, had consequently been subjected to an appreciably greater tonnage of traffic.

The position in the ingot of these rails was such that they should have been free from any segregation effect incident to the too-small hot tops.

Endurance tests were made on these rails.

Results of tests on the D rail from ingot 13 of heat No. 1 have been included in Figure 15. These data also show appreciable scatter but indicate quite definitely an endurance limit at approximately 43,000 lbs./in.$^2$, slightly lower than that indicated by the second group of specimens from the B rail from the same ingot.

The "scatter," however, tends to show that the marked scatter shown by the B rail from the same ingot was not essentially related to the use of too small a hot top, but is apparently characteristic of the steel from this ingot.

The results of tests on specimens from the E rail of ingot No. 21 of heat No. 3 are given in Figure 16 (*b*). Except for the Y specimens, the scatter is small. The endurance limit is very definitely indicated at 52,500 lbs./in.$^2$, slightly greater than that shown by the second group of specimens from the B rail of the same ingot.

The results of all tests made on the Canadian Pacific rail have been plotted in Figure 27.

A comparison of these data with the similar data for the Baltimore & Ohio rail (fig. 7) emphasizes the very marked "scatter" of results of tests from the Canadian Pacific rail due to the presence of internal cracks in the rails.

A very careful examination was made of sections from rails L–15, L–17, and H–15 returned from service on the Baltimore & Ohio Railroad to determine if shatter cracks were present in these rails. That none were found tends to further confirm the conclusion that "scatter" in the Canadian Pacific rails was due to the presence of internal cracks.

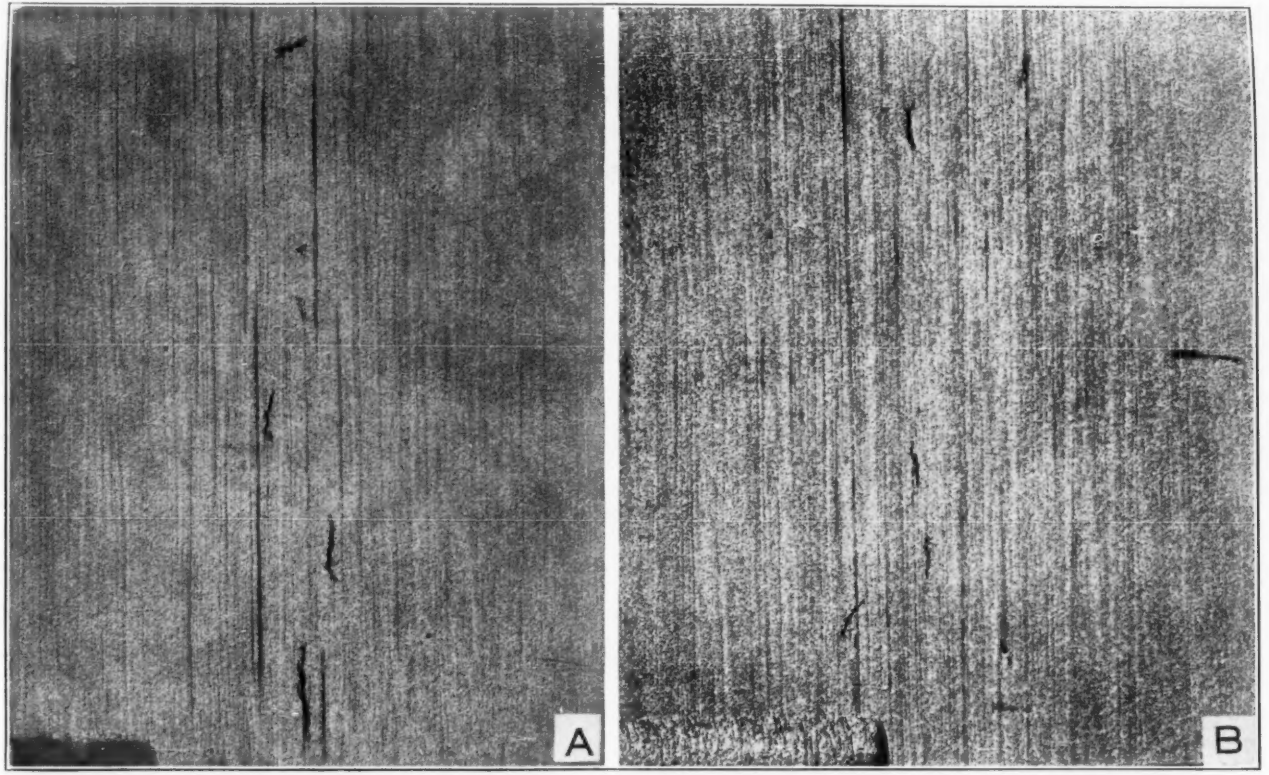


FIGURE 26

*A* and *B*, Cracks in new B rail from heat No. 3, ingot No. 11, etched in hot concentrated HCl. × 1

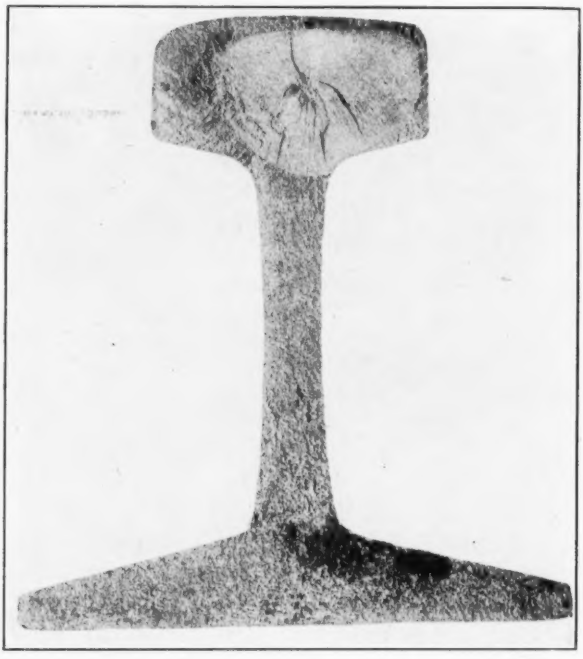

FIGURE 28.—*Transverse fissure in rail CN1 which failed in service*

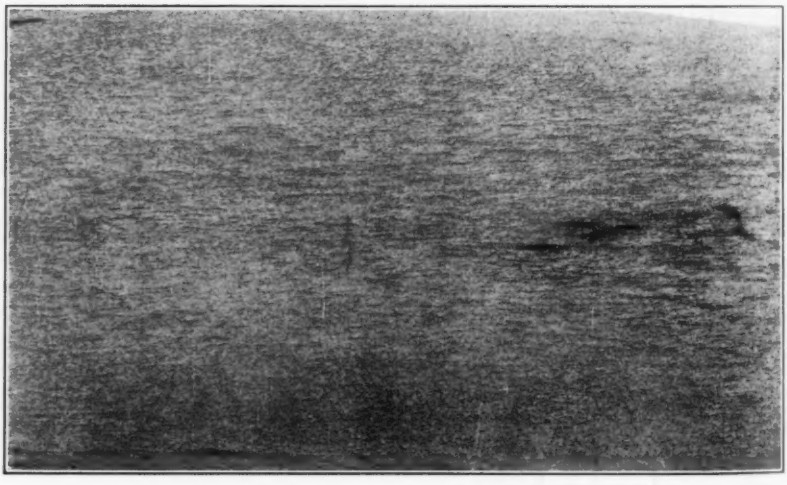

FIGURE 30.—*Internal cracks in fissured rail (CN1)*

Deep etched in hot HCl. × 1

The ability of some of the specimens tested at the lower stress values to withstand repeated stress at appreciably higher values is also indicated in Figure 27.

It must be borne in mind in considering the results of "restressing" tests that the apparent improvement in stress values is not necessarily entirely the result of the "understressing" of the test specimen but includes the difference in stress between the endurance limit of the specimen and the lowest stress at which the test was started.

Several of the specimens on which the tests were started at or slightly above 40,000 lbs./in.$^2$ might, therefore, have run indefinitely

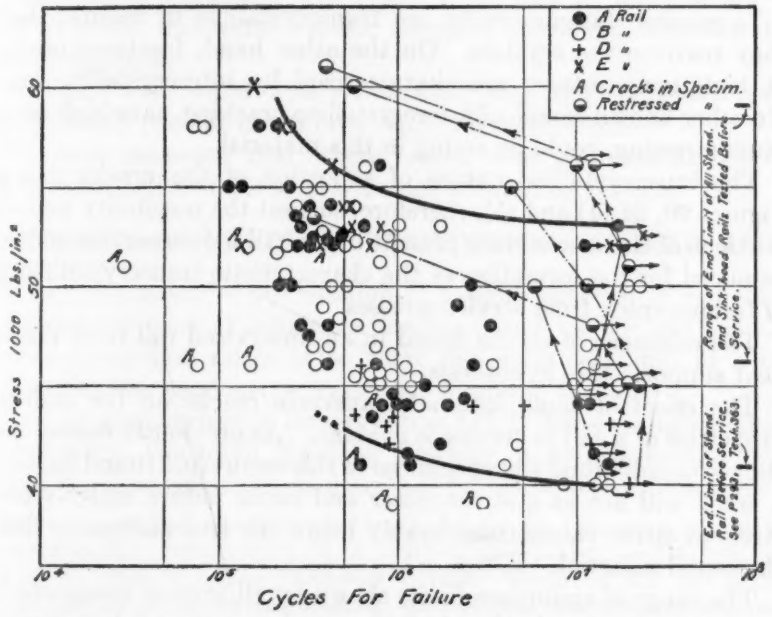


FIGURE 27.—*Results of all endurance tests on steel from rails returned from service on the Canadian Pacific Railway*

at appreciably higher stresses. This is quite evident from the test of the specimen from the D rail of ingot No. 13 at 40,000 lbs./in.$^2$ (Fig. 15.) Another specimen from this rail ran at 43,000 lbs./in.$^2$ without failing. The fact, then, that the specimen that ran at 40,000 lbs./in.$^2$ when restressed also ran without failure at 45,000 lbs./in.$^2$ indicates a known definite increase from the understressing of only 2,000 lbs./in.$^2$

In the previous report there was evidence of appreciable "scatter" of the test results, notably in the Y specimens from the B rail of heat No. 3, as indicated in Figure 8 of the previous report (p. 281).

The end of one of these broken specimens from an unused rail that had failed prematurely was cut longitudinally, ground and

polished. Deep etching revealed the presence of internal cracks, as illustrated in Figure 25. In view of this, a section of the head of the B rail from ingot No. 11 of this heat which had not been in service [7] was critically examined by slicing, grinding, and deep etching longitudinal surfaces parallel to the tread.

Evidences of cracks in this new rail were found and are shown in Figure 26 (*a*) and (*b*).

This evidence suggests that the cracks found in the rails after service may have been present in the rails before service. In the micrographs (figs. 20, 23, and 24 (*a*) and (*b*)) attention was directed to the intercrystalline nature of portions of the cracks.

In general, fatigue cracks are transcrystalline in nature; that is, they traverse the crystals. On the other hand, fractures produced at high temperatures are characterized by intercrystalline cracks. No other known source of intercrystalline cracking, save high temperature stressing, could be acting in this material.

The intercrystalline nature of a portion of the cracks shown in Figures 20, 24 (*a*) and (*b*), therefore, suggest the possibility that these portions of the cracks were present in the rail before service and acted as nuclei for the formation of the characteristic transcrystalline type of fatigue crack from service stresses.

The existence of cracks found in an unserviced rail from the same heat supports this hypothesis.

The effect of small internal transverse cracks on the endurance properties of a rail in service is evident. As previously stated and as shown for individual test specimens in this report (p. 210 and fig. 16 (*a*)), a crack will act as a stress raiser and cause failure under repeated stress at stress values considerably below the true endurance limit of the sound material.

The range of endurance limits shown by all tests of specimens from new rail before service in which the specimens were taken across the section of the head has also been indicated in Figure 27.

It is evident that in many cases the endurance limit of the rail steel after service is apparently lower than the lowest value indicated by the new rail of 46,000 lbs./in.$^2$; the lowest limit indicated by the serviced rail is apparently approximately 42,000 lbs./in.$^2$

It is of interest to note, however, that the endurance limit of steel from a new standard rail in which the specimens were taken contiguously from the U position was found to be about 41,000 lbs./in.$^2$, as shown in Figure 21, page 293, of the previous report, and indicated in Figure 27 of the present report.

Apparently, then, the lower limit of endurance indicated by the serviced rails is not less than may be present in some new rail.

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[7] See previous report (B. S. Tech. Paper No. 363, p. 332, Table 13, footnote 2) for details of this rail.

### 2. SUMMARY OF RESULTS OF TESTS ON CANADIAN PACIFIC RAILS

A series of endurance tests have been made on specimens taken from 10 rails from 2 special heats which had been subjected to more than 12,000,000 tons of traffic in track of the Canadian Pacific Railway.

Similar tests had been made on new rails from the same heat before service so that the effect of service on the endurance properties could be determined.

The results of the endurance tests showed considerable "scatter," indicating marked inhomogeneity in the steel.

This marked "scatter" was found to be due to the presence of small internal transverse cracks, so-called shattered zones in the head of the rails.

Evidence was found which indicated that the shattered zones were probably present in the rails before laying in track.

The data indicated that where cracks were not present the endurance properties of the steel were not affected by service.

## VI. RESULTS OF TESTS OF TRANSVERSE-FISSURED RAILS

In any discussion of the relation of fatigue strength of rail steels to the development of transverse fissures in service the obvious and important question arises as to what the endurance limit of a rail steel is that has displayed a transverse fissure in service.

Through the courtesy of Robert Job and C. B. Brown, chief engineer of operations, Canadian National Railway, a 10-foot section from each of the following rails, as described by Mr. Job, was obtained:

*Rail No. 1.*—Failed, due to internal transverse fissure. Manufactured by the Algoma Steel Co., 100-pound A. R. A., 1919–4, open hearth, section 120, heat 6169–19–B. Date of failure, December 4, 1927. Weight of rail, 100 pounds per yard; length of rail, 33 feet; location, 800 feet from mile board 10, Cornwall subdivision, St. Lawrence division, section 2. This rail broke due to transverse fissure.

*Rail No. 2.*—An adjoining rail removed for your examination. No fissure had been found in this rail. This also was a 100-pound section made by Algoma Steel Co., A. R. A., April, 1919, open hearth, heat 8148–1–A.

The rails were taken from adjoining positions in track and were said to have been subjected to the same conditions of service.

A photograph of the fissure is shown in Figure 28. It is also noteworthy that there is no marked flow of the top surface of the head of the rail as was apparent in rail L–15. (Fig. 8.)

Through the courtesy of E. A. Sperry, president, Sperry Development Co., both rails were examined in his laboratory with the Sperry rail flaw detector [8] for internal cracks.

[8] See footnote 4, p. 211.

The results of the examination were reported by Mr. Sperry as follows:

> There are no fissures of the size of 1 per cent of the area of the head, or larger. A 1 per cent fissure is the smallest that we can locate with the facilities now at hand.

Mr. Sperry was requested to make the examination with the expectation that other fissures might be located and the endurance-test specimens then taken adjacent to them in order to determine the endurance properties of the steel in the rail at a section adjacent to internal cracks, incomplete fissures.

In view of the apparent freedom of the rail from other internal cracks, 12 fatigue-test specimens were cut from the fissured and unfissured rails in the usual manner (fig. 2) and their respective endurance limits determined. In the unfissured rail these were taken at a section approximately 8 feet from end of rail; in the fissured rail about 8 feet from the fissure.

The results of these endurance tests are given in Figure 29.

It is evident that the endurance limit of the steel from the fissured rail of 56,800 lbs./in.$^2$ is appreciably greater than the value of 44,200 lbs./in.$^2$ shown by the steel from the unfissured rail. In fact, the value of approximately 57,000 lbs./in.$^2$ shown by the steel from the fissured rail approaches the maximum limit of endurance values of 59,000 lbs./in.$^2$ (fig. 7) found for any of all the rail steels tested, and the value of approximately 44,000 lbs./in.$^2$ is nearer to the lower limit of endurance values of approximately 41,000 lbs./in.$^2$ (fig. 27) found for all rail steels tested.

It is apparent from the photograph (fig. 28) that the nucleus of the fissure is near the center of the head of the rail. In view of the data given from the tests of specimens from rail L–15 (p. 222) returned from service on the Baltimore & Ohio Railroad, it is certainly improbable that the nucleus of this fissure originated in service as a result of service stresses, especially when the relative freedom of surface flow of the head of the fissured rail is considered.

A section approximately 4 inches long of the fissured rail, about 7 feet from the fissure, was cut diagonally, as indicated in Figure 21, and the inner surfaces examined for internal cracks by deep etching in hot hydrochloric acid. Evidence of internal failure, shatter cracks, and an incipient fissure, was found as shown in Figure 30. These cracks were located approximately in the center of the head of the rail in the same relative location as the nucleus of the fissure (fig. 28).

The depth of these cracks was small, probably less than one-eighth inch, as measured by grinding off the surface and reetching until the larger ones disappeared. This indicates that the area was probably less than the limit of sensitivity of 1 per cent of the Sperry

rail flaw detector, which explains their presence not being detected by this instrument.

A similar section taken from the rail immediately behind the fissure showed similar, but smaller, cracks.

The endurance-test data were uniform in both cases, showing relatively little scatter except for the Y specimens; in one from the fissured rail and in both from the unfissured rail. In each case the specimen broke at approximately from one-fourth to one-half inch distance from the minimum section, indicating marked inhomogeneity.

The broken ends were deep etched in hot hydrochloric acid. That one from the fissured rail showed the presence of minute cracks in

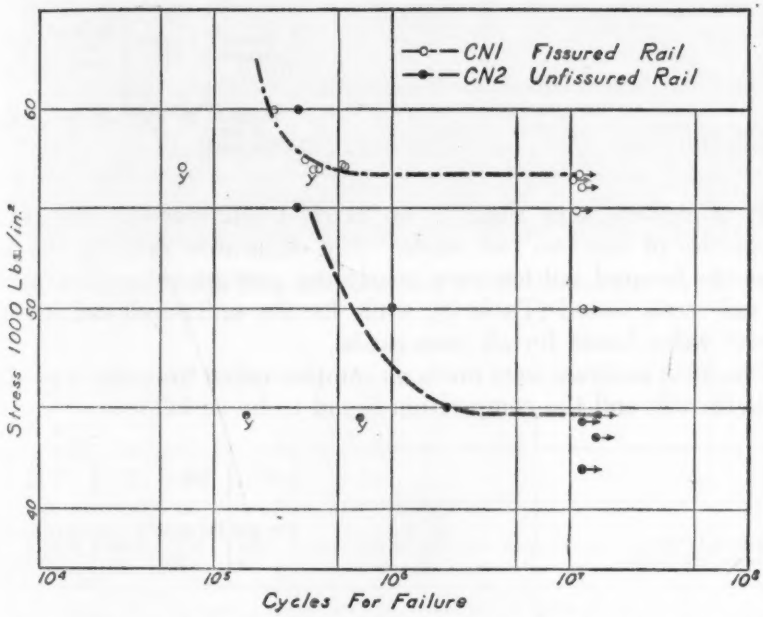


FIGURE 29.—*Endurance curves of steel from fissured rail (CN1) and unfissured rail (CN2) after same service*

the heavier sections, indicating early failure was probably due to the presence of a crack similar to those indicated in Figure 30.

Cracks were not evident in the specimens from the unfissured rail, and deep-etched sections of this rail did not indicate their presence. In one section, however, an unusually heavy segregated streak was apparent, as shown in Figure 31. This section was cut from the rail adjacent to the sections from which the endurance-test specimens were taken. The segregated streak occupied a position in the head approximately contiguous with that from which the Y specimens were taken. On deep etching the broken ends of the specimens a

similar segregated streak was apparent. Due to the curvature of the specimen, a satisfactory photograph could not be obtained. However, the fractured end of one of the specimens was ground flat and deep etched. The end of the segregated streak was apparent, as shown in the photograph in Figure 32. This section was approximately one-thirty-second inch back of fracture.

Undoubtedly the early failure of these Y specimens from the unfissured rail was due to the presence in the test sections of the segregated streaks.

Tensile properties of specimens from the O position of the fissured and unfissured rails were determined, using standard 0.505-inch diameter test specimens, and were found to be as given below.

| | Tensile strength | Elongation | Reduction of area | Endurance ratio |
|---|---|---|---|---|
| | *Lbs./in.²* | *Per cent* | *Per cent* | *Per cent* |
| Fissured rail (CN1) | 136,000 | 11.5 | 14.5 | 41.7 |
| Unfissured rail (CN2) | 130,000 | 12.0 | 16.0 | 34.0 |

It is evident that there is no marked difference in the tensile properties of the two rail steels. The endurance ratio of the steel from the fissured rail has very nearly the average value obtained for all rail steels tested (Table 3), while for the unfissured rail it is the lowest value found for all tests made.

Chemical analyses were made on samples taken from the O position of both rails and the composition found to be as follows:

| | C | Mn | P | S | Si |
|---|---|---|---|---|---|
| | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* |
| Fissured rail (CN1) | 0.85 | 0.72 | 0.030 | 0.040 | 0.13 |
| Unfissured rail (CN2) | .75 | .73 | .030 | .030 | .17 |

The relatively high endurance limit of the steel from the fissured rail, the relatively uniform results of the plotted test data for both the fissured and unfissured rail steels when free from cracks or inclusions, the fact that both rails were submitted to the same traffic, the presence of shatter cracks in the fissured rail and their absence in the unfissured rail, all tend to indicate that the transverse fissure, admittedly a fatigue failure, must have started from a preexistent nucleus.

Through the courtesy of Dr. M. E. McConnell, chief chemist, Pennsylvania Railroad, a rail was obtained which had failed due to a transverse fissure after the relatively short service of five months. This was a 130-pound P. S. rail from Bethlehem Steel Co. heat No.

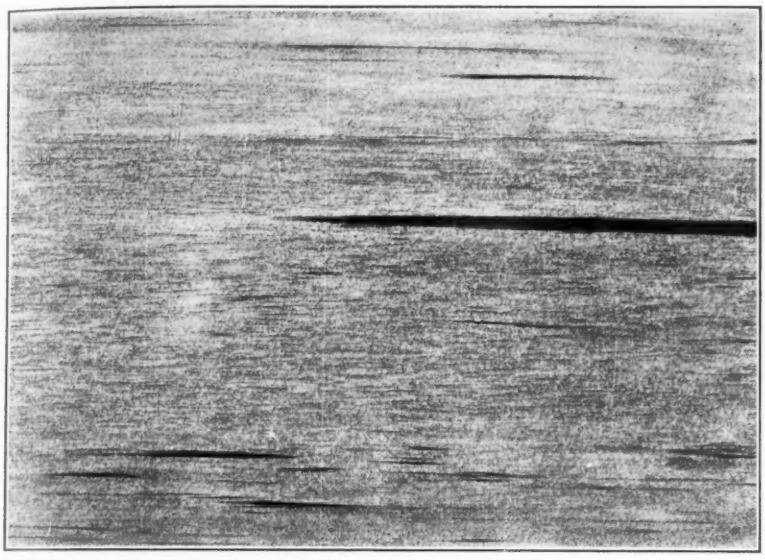

FIGURE 31.—*Section from unfissured rail (CN2) showing segregated streak*

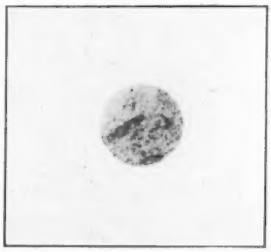

FIGURE 32.—*Section of fatigue specimen (CN2–Y1) about one thirty-second inch behind fracture showing section of segregated streak similar to that shown in Figure 31 from same rail*

Deep etched in HCl. × 3

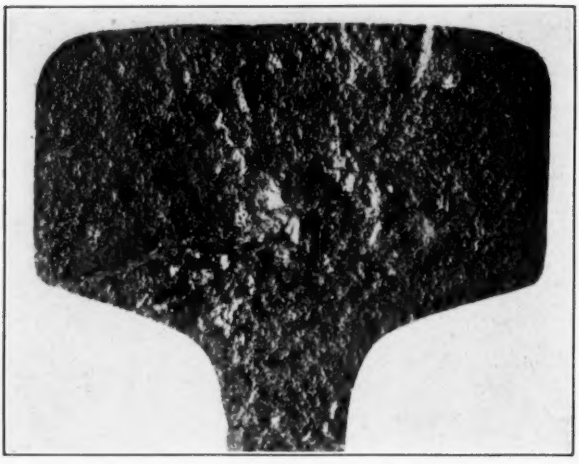

FIGURE 33.—*Transverse fissure in rail PO which failed in service.* × *1*

FIGURE 34.—*Internal cracks in fissured rail PO*

Deep etched in hot HCl. × 1



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FIGURE 36.—*Portion of crack shown in Figure 29.* × *500*

Arrow indicates probable intercrystalline portion



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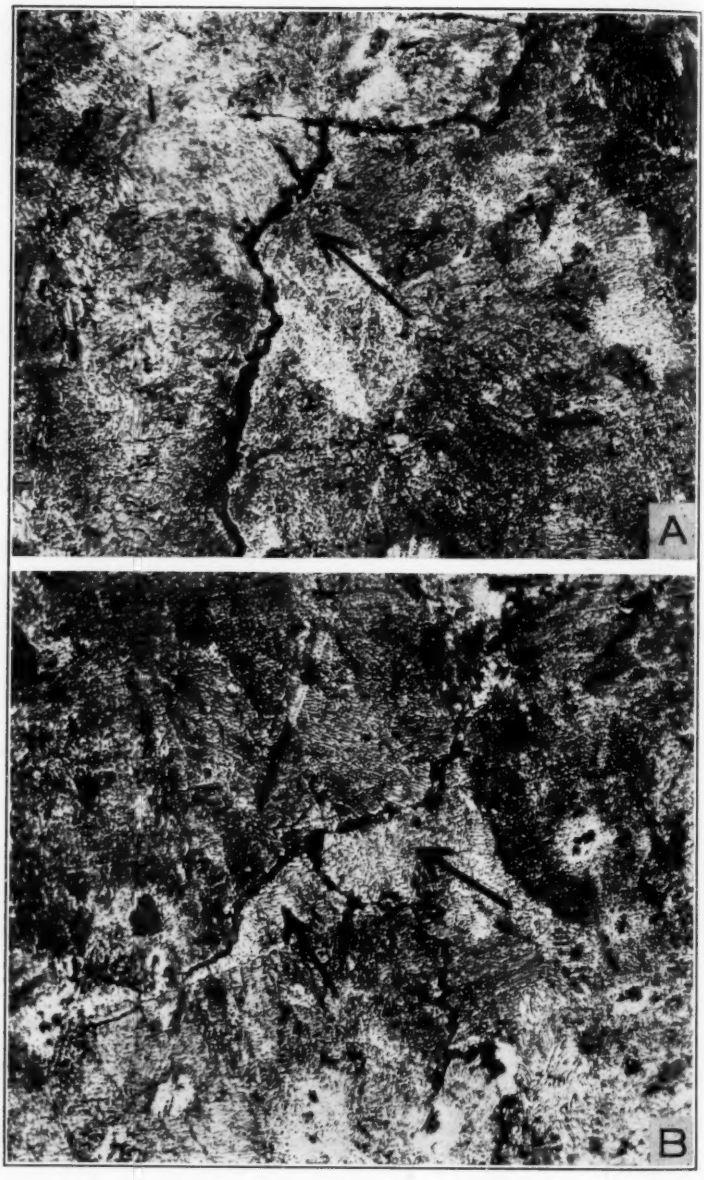


FIGURE 37.—*A, B, portions of cracks shown in Figure 23.* × *500*

Arrows indicate probable intercrystalline portions

72284–8–C. Cushing [9] has reported a detailed study of similar failures of rails from the same ingot and heat in which particular attention was directed to the presence of numerous shatter cracks.

A photograph of the fissure is given in Figure 33. Due to the relatively short life of the rail, the characteristic polished surface usually found in rails failing in a similar manner after prolonged service was not developed.

A longitudinal section of this rail taken approximately between 1 to 5 inches from the fracture was examined for internal cracks by

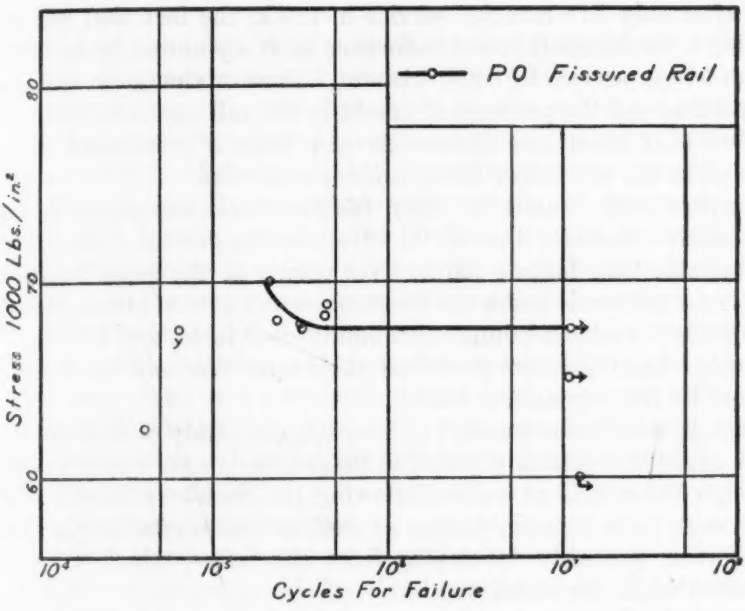


FIGURE 35.—*Endurance curve of steel from fissured rail PO*

deep etching in the same manner as previously described. Well-defined internal fissures were found, as is evident in Figure 34.

Endurance tests were made on 12 specimens taken from this rail in the usual manner. (Fig. 2.) The results are given in Figure 35. It is evident that there is marked "scatter" of the test data, which was found to be due to the presence of internal cracks. A definite endurance limit for the steel, however, was indicated at approximately 67,500 lbs./in.$^2$ This is the highest value for the endurance limit of a rail steel indicated by any of the steels from either serviced or new rails presented in this and the previous report.

[9] Cushing, W. C., Rail and Wheel, Pt. III, The Genesis of the Transverse Fissure, Bull. Am. Rwy. Eng. Assoc., **30**, No. 315, p. 227; March, 1929. See particularly p. 268.

The tensile properties were found to be as follows: Tensile strength, 148,000 lbs./in.$^2$; elongation, 8.5 per cent; and reduction of area, 14 per cent.

The endurance ratio is then 45.5 per cent, or slightly higher than the average.

Chemical analysis of a specimen from the O position showed the composition to be carbon, 0.82 per cent; manganese, 0.82 per cent; phosphorus, 0.033 per cent; sulphur, 0.04 per cent; and silicon, 0.24 per cent.

The unusually high endurance limit of this steel, the failure of the rail after only five months' service in track, the fact that rail steels having a considerably lower endurance limit are known to have given much longer service in track without failure or change in endurance properties, and the presence of cracks in the rail again favor the conclusion that transverse fissures develop from a preexistent nucleus, present in the rail before being subject to service.

As previously stated (p. 236), fatigue cracks are generally transcrystalline in nature; that is, they traverse the crystal. No evidence is available that fatigue failure ever occurs at the grain boundaries in ferrous materials under the frequencies of repeated stress, generally below 2,000 cycles a minute, commonly used in fatigue testing. On the other hand, fracture produced at elevated temperatures is characterized by intercrystalline cracks.

Waring and Hofammann,[10] in their original study of shatter cracks, state that "the defects (cracks) in fissured rails were found to extend through the crystal as well as following the boundary lines"; that is, they were both transcrystalline as well as intercrystalline in nature, the former probably developing from the latter, which must have occurred while the metal was hot.

Detailed microexamination was made of the shatter cracks found in the transverse fissured rails CN1 and PO to determine if intercrystalline failure was present. The locating of any intercrystalline nucleus of a transcrystalline fatigue crack is of necessity rather doubtful, in fact largely a question of chance. The intercrystalline nucleus which is being sought is of miscroscopic dimensions and may quite possibly be destroyed and entirely lost during the necessary cutting, grinding, and polishing operations incident to the preparation of the specimen for miscroscopic examination. Rail CN1 had a carbon content very close to the eutectoid composition. The structure showed no free ferrite at the grain boundaries, which made it very difficult to determine boundaries between the pearlite grains. The micrograph shown in Figure 36 illustrates the general appearance of a portion of the crack shown in Figure 30. Along the section indicated it appears to be intercrystalline, but the evidence is not

[10] Waring and Hofammann, Deep Etching of Rails and Forgings, A. S. T. M., **19**, Pt. II, p. 183; 1919.

conclusive. The difficulty of determining grain boundaries in a steel of this structure is quite apparent.

Similarly, the structure of the steel in the fissured rail PO had no free constituent, making it difficult to determine the path of the fatigue crack, whether across or between the grains.

Figure 37 (*a*) and (*b*) show characteristic portions of one of the cracks shown in Figure 34, which at points indicated are apparently intercrystalline.

## VII. SUMMARY AND CONCLUSIONS

A series of tests has been carried out to determine the effect of service on the endurance properties of rail steels.

In a previous report, data were given on the endurance properties of steels from new rails from 11 distinct heats. An hypothesis based on the phenomena of weakening by "overstressing" and strengthening by "understressing" was presented whereby it was considered possible to determine whether the steel of the rails in service was subjected to stresses greater or less than its endurance limit by a comparison of the endurance limits before and after service.

Through the cooperation of the Baltimore & Ohio Railroad and the Canadian Pacific Railway, a total of 18 rails from 6 heats tested before service and 1 heat not previously tested were removed from track after being subjected to over 20,000,000 and 12,000,000 tons of traffic, respectively. These were principally A and B rails.

The endurance properties were determined in the same manner as for the rail steels before service.

In the rails returned from service on the Baltimore & Ohio Railroad the endurance properties of the steel were found to be the same as before service.

Tests were also made to determine the effect of appreciable surface flow and consequent hardening of the top of the head of the rail in service on the endurance properties of the steel in that rail. No evidence was found of any weakening by "overstressing" or strengthening by "understressing" having occurred in the steel immediately below the work-hardened layer.

Comparative tests were also made on steel from a rail which failed in service due to a transverse fissure and a similar rail from an adjoining position in track which had been subjected to identical service but had not failed.

It was concluded that under the service conditions to which the rails were subjected on the Baltimore & Ohio Railroad the steel in the head of the rail was not subjected to fatigue stresses greater than its endurance limit.

The results of similar tests of rails after service on the Canadian Pacific Railway were not so definite in their indications. In many cases the test specimens showed a very marked "scatter" in endurance properties, indicating appreciable inhomogeneity in the steel. This "scatter" was found to be due to the presence of groups of minute transverse cracks, or so-called "shattered zones," in the rail as removed from service.

Microscopic examination showed the cracks to be principally transcrystalline, but in a few instances evidence was found of intercrystalline cracking which would not have been produced by alternating stress and therefore must have been formed in the metal while hot, before it was put into service.

Examination of a new rail from one of the same heats showed the presence of shatter cracks. Premature failure of a fatigue-test specimen from a new rail of the same heat was also found to be due to the presence of an internal crack.

Unless cracks were present the steel from rails after service gave, in most cases, endurance limits within the range of those obtained on steel from new rails.

Cracks were not found in the fatigue specimens or the etched sections of rails removed from track on the Baltimore & Ohio Railroad which had been subjected to over 20,000,000 tons of traffic as compared to about 12,000,000 tons for the Canadian Pacific rail, and, as previously stated, no detectable change in the endurance properties of the former was found. The conclusion was unavoidable either that the service on the Baltimore & Ohio rails had not created nuclei for fatigue failure or that the test sections of the endurance-test specimens did not happen to include such nuclei. Certainly no general deterioration of endurance properties occurred as a result of the service to which these rails were subjected or it would have become evident by a more general scatter of the test results.

The conclusion seems justifiable that the service stresses imposed on the Baltimore & Ohio rails were below the endurance limit of present-day rail steels containing the usual amount of nonmetallic inclusions or other internal defects but free from transverse cracks.

The endurance limits of the steel from the used Canadian Pacific rails, while in general within the limits found for the steel from new rails, did show in some cases an appreciably lower endurance limit than the steel from new rails from the same ingot position and heat, although these rails had carried less than 60 per cent as much traffic as the rails from the Baltimore & Ohio.

The scatter of the points on the endurance curves was greater for steel from the used than from the new Canadian Pacific rails from the same heats, and several early failures of test specimens were definitely correlated with cracks in the rail.

Transverse internal cracks, logical nuclei for fatigue failures, were found in the used rails, evidence that detectable deterioration of the rail may have occurred in service.

If there were evidence also that the Canadian Pacific rail had been subjected to track stresses greater than those imposed on the Baltimore & Ohio rails and if it could be shown that the new rails were free from internal cracks or other stress raisers, the results of tests on the steel from the Canadian Pacific rail would tend to support the theory that internal cracks and subsequent transverse fissures may originate in service.

However, the fact that transverse cracks were found in new rail from the same heats and the test results of the steel from the Baltimore & Ohio rails which had been subjected to over 60 per cent more traffic support far more strongly the theory of a preexistent nucleus (shatter crack) in the rail before placing in service, with the subsequent development of the characteristic fatigue failure of the transverse fissure.

The results of tests on specimens from a transverse-fissured rail and a comparison rail in which it was found that the endurance limit of the steel in the fissured rail was appreciably greater than that in an adjacent rail in track subjected to the same traffic confirms the conclusion that a transverse-fissure failure is not due simply to fatigue stresses developed in track. If this were the case, the rail steel having the lower endurance limit should have failed first. The fissure must have progressed from a stress raiser originally present in the rail. The presence of internal transverse cracks, as shown by deep etching in the fissured rail, and absence of similar cracks in the unfissured rail tend to show that one or more of these minute cracks acted as the stress raiser or nucleus of the transverse-fissure failure.

The absence of deterioration in the used Baltimore & Ohio rails with the absence of cracks in the Baltimore & Ohio rails, and the presence of deterioration in the used Canadian Pacific rails with the presence of cracks in the new Canadian Pacific rails, and the absence of cracks in the unfissured and presence of cracks in the fissured rail from the Canadian National Railroad, all appear to corroborate the statement of Moore and Kommers:[11] "Transverse fissures seem to develop from some defect which is initially in the rail and which acts as the nucleus of a fatigue failure."

Whether or not this conclusion will ultimately be proved and accepted, the evidence for it is sufficiently strong to justify careful search for the metallurgical factors which tend to produce or to prevent the presence of initial defects which may develop into transverse fissures. Experimental work on one phenomenon met in rail

[11] Moore, H. F., and Kommers, J. B., The Fatigue of Metals, p. 240; 1927.

steel at high temperatures, which may have a possible bearing on the formation of initial defects, is now in progress at the Bureau of Standards.

## VIII. ACKNOWLEDGMENTS

The authors wish to express their appreciation and thanks to E. Stimson, chief engineer of maintenance of the Baltimore & Ohio Railroad; J. M. R. Fairbairn, chief engineer of the Canadian Pacific Railway; Robert Job and the Canadian National Railway for their indispensable cooperation; and to Dr. H. W. Gillett, chief, division of metallurgy of the Bureau of Standards, for many helpful suggestions throughout the course of the investigation.

WASHINGTON, March 22, 1929.

# THE STRUCTURE OF ∝ METHYLXYLOSIDE

By F. P. Phelps and C. B. Purves

ABSTRACT

Starting with pure $\beta$ methylxyloside, trimethyl $\beta$-methylxyloside has been prepared, purified and hydrolyzed, giving $\alpha$-trimethyl xylose which has previously been shown to have an amylene-oxide ring structure. Although previous investigators have failed to obtain crystalline trimethyl xylose by the hydrolysis of trimethyl $\alpha$-methylxyloside, it is here shown that by starting with very pure $\alpha$-methylxyloside, methylating the same by the silver oxide method and carefully purifying the resulting sirupy trimethyl $\alpha$-methylxyloside, the latter on hydrolysis gives in good yield (85 per cent) the same crystalline trimethyl xylose as was obtained from the $\beta$-isomer. It is therefore definitely proved that the two trimethylxylosides and likewise the two methylxylosides have the same ring structure.

The rotations of the various compounds have been measured after careful purification and found to differ considerably from those previously recorded.

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## I. DISCUSSION OF METHYLATION WORK, PAST AND PRESENT, ON XYLOSE AND ITS BEARING ON STRUCTURE

Crystalline trimethyl xylose, derived from the corresponding $\beta$-methylglycoside, was the first member of the methylated monosaccharides to which an amylene-oxide ring structure could be assigned with certainty,[1] and at the time the opinion was expressed that not only $\alpha$- and $\beta$-methylxyloside, but possibly xylose itself, were similarly constituted. The intervening years, however, have brought to light several cases in which methylating agents, applied to a reducing sugar, have produced derivatives of more than one ring type, and in this connection the behavior of galactose and arabinose may be cited.[2] Pryde, Hirst, and Humphries have shown

[1] Hirst and Purves, J. Chem. Soc., **123**, p. 1352; 1923.
[2] Pryde, Hirst, and Humphries, J. Chem. Soc., **127**, p. 348; 1925.

that in the case of these two sugars methylation with methyl sulphate and alkali undoubtedly gives at least two ring forms. In the case of xylose, also, this method gives an impure product, the rotation of the methylated products being too low in addition to the yield being small. Our own values obtained through the use of this method are in approximate agreement with those of Carruthers and Hirst,[3] using the same method, and are much lower than the values obtained when pure $\alpha$- or $\beta$-methylxyloside was methylated with silver oxide and methyl iodide. Likewise it is now well known that the condensation of a reducing sugar with acid methyl alcohol, if carried out at room temperature, yields a $\gamma$-methyl glycoside, while at higher temperatures, such as Fischer originally used, a mixture containing largely the $\alpha$ and $\beta$ forms is obtained.[4] Our own work indicates that all three are formed at an intermediate temperature and that the equilibrium reached in acidified methyl alcohol is not an equilibrium between the $\alpha$ and $\beta$ forms only, but between the $\alpha$, $\beta$, $\gamma$, and perhaps, other forms as well, the predominating form being determined by the conditions, of which temperature is particularly important. As only two of these three forms can have the same ring structure, the third must have a different ring, and there is no known way to tell which, if either, of these rings exists in the parent sugar. It accordingly follows that any conclusions regarding the identity of ring structure in the sugar series to be satisfactory must be based upon nothing less than a direct experimental identification and correlation of the derivatives concerned. Such a correlation has now been made for $\alpha$- and for $\beta$-methylxyloside.

The earlier methylations of this sugar were carried out by Carruthers and Hirst,[3] who used methyl sulphate and caustic soda to prepare the $\beta$ form and Fischer's method, to condense xylose with methyl alcohol for the $\alpha$ form. In the latter case, after that portion of the $\beta$-methylxyloside which crystallized from the product had been removed, the residual crude sirup was treated with silver oxide and methyl iodide and gave a mobile liquid with the elementary composition of a fully methylated pentose. When heated in acid methyl alcohol, this liquid displayed a final specific rotation of 50.4°, a value in good agreement with that of 49.5° acquired by trimethyl $\beta$-methylxyloside under similar circumstances. Such behavior has long been considered to be characteristic of an $\alpha$-$\beta$ pair of methyl glycosides possessing the same ring structure,[5] and the reasonable inference was made that one more example of the same phenomenon had been encountered. While, therefore, it is true that trimethyl $\alpha$- and $\beta$-methylxyloside have been correlated, the method used was an

[3] Carruthers and Hirst, J. Chem. Soc., **121**, p. 2299; 1922.
[4] See also Fischer, Ber., **47**, p. 1980; 1914.
[5] Purdie and Irvine, J. Chem. Soc., **104**, p. 9; 1904.

indirect one, and a series of considerations led the present authors to doubt its reliability in this case. Thus, the methylated glycosides examined by the former workers did not conform to the decisive test of similar ring structure devised by Purdie and Irvine [5] in that trimethyl β-methylxyloside, when hydrolyzed with aqueous acid, yielded a crystalline trimethyl xylose, $[\alpha]_D = 20°$ in water, while the impure α isomer gave rise to a lævorotatory sirup. Moreover, the correlation adopted by Carruthers and Hirst is not valid if two trimethyl xyloses, differing in ring type, happen to possess similar equilibrium rotations in acid methyl alcohol; a possibility which can not be counted remote when it is remembered that Haworth and Westgarth [6] have recently prepared a second trimethyl xylose with a specific rotation of 31.2° in water, or within 12° of the former figure. Thirdly, in the preparation of trimethyl β-methyl xyloside from xylose by the methyl sulphate and sodium hydroxide method there is some uncertainty because of the small yield obtained, together with the suspicion that this method does not always yield a single product. If the trimethyl β-methylxyloside contained a large unknown percentage of a second compound of similar properties and, therefore, difficult to separate and, as in this case, on hydrolysis yielded less than 50 per cent of the theoretical amount of trimethyl xylose, it would be difficult to say which of the two components in the parent mixture produced the trimethyl xylose. A fourth consideration lay in the fact that α-methyl glucoside, methyl-galactoside, and methyl-arabinoside are directly correlated, in each case, to methylated derivatives which very probably possess an amylene oxide ring.[7] It follows that a similar structure must be assigned to the glycosides themselves, and if this is so, a computation in molecular rotations by the method of Hudson [8] leads to the conclusion that the 1:5 oxygen linking is not present in α-methylxyloside. As this is at variance with the accepted structure for the β-isomer, Dr. C. S. Hudson requested us to repeat the work of Carruthers and Hirst and to obtain more decisive experimental evidence on the matter.

The carefully purified crystalline α-methylxyloside was methylated by the silver oxide reaction, and every care was taken to render the methylation complete. Although the trimethyl α-methylxyloside could not be made to crystallize and purification was carried out by the uncertain method of distillation, the specific rotations determined, 121.7° in chloroform, 112.7° in water, and 122° in methyl alcohol, are regarded as approximately correct. When methylated in similar fashion β-methylxyloside gave an excellent yield of the crystalline,

[5] Purdie and Irvine, J. Chem. Soc., **104**, p. 9; 1904.
[6] Haworth and Westgarth, J. Chem. Soc., **129**, p. 880; 1926.
[7] Charlton, Haworth, and Peat, J. Chem. Soc., **129**, p. 89; 1926. Hirst, J. Chem. Soc., **129**, p. 350; 1926. Hirst and Robertson, J. Chem. Soc., **127**, p. 358; 1925. Haworth, Ruell, and Westgarth, J. Chem. Soc., **125**, p. 2468; 1924.
[8] Hudson, J. Am. Chem. Soc., **48**, p. 1434; 1926.

fully methylated derivative. Purification was rigorous, and the final constants were m. p., 51°; $[\alpha]_D^{20} = -69.5°$ in chloroform, and −81.7° in water. Aqueous acid, of a lower concentration and at a lower temperature than that previously employed, hydrolyzed both trimethyl methylxylosides, 80 to 85 per cent of the theoretical yield of crystalline trimethyl xylose being isolated in each case. The two specimens of the sugar were then compared with each other and found to be identical. Mutarotation in water was swift, the initial specific rotation being 65° and the equilibrium rotation 17.7° in place of the value of 20° hitherto accepted as standard. Trimethyl xylose occasionally displayed mutarotation in chloroform solution, the initial value of $[\alpha]_D^{20} = 58°$ diminishing to 24°; also the initial rotation in this solvent was quite erratic, lying anywhere between 58 to 45 after recrystallization from ether solution. In these cases it is possible that the sugar crystallized as a mixture of the $\alpha$ and $\beta$ forms. The value of 58° is, therefore, to be taken with caution. One other point remained to be considered, for trimethyl $\beta$-methylxyloside prepared by the methylation of xylose in alkaline solution had a specific rotation some 10° lower than the correct value of −69.5° in chloroform. The similar observation of Carruthers and Hirst was thus confirmed, and recrystallization failing to improve the constant, the possibility remained that the two methods of alkylation had produced isomeric substances. On hydrolysis, however, the specimen with the incorrect specific rotation also gave trimethyl xylose in 75 per cent of the theoretical amount, and the discrepancy was seen to be due to an impurity, possibly an isomeric form, which was difficult to remove. This observation should be of value when it is desired to prepare a methylated sugar in a state of optical purity by the aid of methyl sulphate and of alkali.

## II. EXPERIMENTAL[9]

### 1. PREPARATION AND PROPERTIES OF TRIMETHYL $\beta$-METHYLXYLOSIDE

(*a*) A 5 g portion of pure crystalline $\beta$-methylxyloside, $[\alpha]_D^{20} = -65°$ in water, was methylated in the standard fashion with 40 g of silver oxide and 25 ml of methyl iodide, a little methyl alcohol being present in this first methylation only to dissolve the glycoside. Acetone was used to recover the product, which was then submitted to a second and a third treatment with the same reagents. The final separation of the trimethyl $\beta$-methylxyloside from the inorganic residues was effected by extraction with ether, the yield of the crude, crystalline material after evaporation of the ether being 5.52 g, or

[9] All methylations with silver oxide and methyl iodide were carried out in a round-bottomed flask equipped with an efficient water-cooled condenser, the two being united by a ground-glass joint in place of the customary rubber stopper. This innovation, while giving a product free from contamination by rubber and, therefore, more readily purified, required care in its use, as flask and condenser readily became cemented together.

88 per cent of theory. After three recrystallizations from petroleum ether, cooled in ice and salt, the specific rotation in purified chloroform was $[\alpha]_D^{20} = -69.6°$ (0.2439 g in 25 ml $CHCl_3$ rotated the plane of polarization 1.36° to the left in a 2 dm tube). This value was not appreciably changed by two further recrystallizations, the value found being $[\alpha]_D^{20}$ in $CHCl_3 = -69.5°$ (0.3669 g in 25 ml gave a rotation of −2.04 in a 2 dm tube). The average of four sets of readings by two observers was $[\alpha]_D^{20}$ in $CHCl_3 = -69.5°$. The rotation in water was $[\alpha]_D^{20} = -81.7°$ (0.2463 g in 25 ml read −1.62 in a 2 dm tube). The substance melted sharply at 51° C. Estimation of methoxyl by Zeisel's method: 0.3144 g gave 1.3968 g of silver iodide, corresponding to 58.7 per cent $OCH_3$. Theory gives 60.2 per cent. A subsequent larger scale preparation (23 g), after purification by several recrystallizations from ordinary absolute ether cooled in ice, yielded substantially the same figures for the specific rotation, namely, −69.4 in $CHCl_3$ and −81.8 in $H_2O$.

(*b*) A 25 g sample of carefully purified xylose was methylated as described by Carruthers and Hirst,[3] first with methyl sulphate and alkali and then with 33 g of silver oxide and 17 ml of methyl iodide. The product crystallized on recovery from the latter process. Yield, 20 g or 64 per cent of the theoretical. Once recrystallized from petroleum ether, 0.0684 g dissolved in 25 ml of chloroform and in a 2 dm tube had an observed rotation of −0.34° and a specific rotation of −59.6°. A second recrystallization failed to change this value (0.1268 g in 25 ml of the same solvent had $\alpha$ −0.60° and $[\alpha]_D^{20}$ −59.0°). It is to be noted that the yield was only 64 per cent as against 88 per cent in (*a*), and likewise that the rotation is about 10° lower.

### 2. PREPARATION AND PROPERTIES OF TRIMETHYL α-METHYLXYLOSIDE FROM PURE α-METHYLXYLOSIDE

Pure α-methylxyloside, prepared by the method of Bourquelot,[10] $[\alpha]_D^{20} + 154.0°$ in water was methylated with 60 g of silver oxide and 37.5 ml of methyl iodide for a 7.5 g sample. Methyl alcohol (2.5 ml) was also present, in this first methylation, to enable the xyloside to dissolve. Three such operations, however, failed to complete the methylation, as was evident when the liquid product was distilled, the refractive index varying with the fraction on which it was observed. Methoxyl, found, 58.4 per cent; theory, 60.2 per cent. Accordingly, the total distillate of 7 g was remethylated with 8 g of silver oxide and 5 ml of methyl iodide. After recovery it was dissolved in water and extracted with chloroform in order to remove most of the lower methylated impurities, and the sirup, obtained from the anhydrous chloroform extract, was methylated for a fifth time. Evaporation at 80°/20 mm, freed the trimethyl methylxyloside from solvent, last traces of which were eliminated during the distillation; bath 110°/10

[3] Carruthers and Hirst, J. Chem. Soc., **121**, p. 2299; 1922.
[10] Bourquelot, Ann. chim., **3**, p. 298; 1915.

mm. Rejecting the first six drops of the distillate, the seventh was found to have $n_D^{24}$ 1.4391 and the last $n_D^{25}$ 1.4387. Since data drawn from the work of Carruthers and Hirst show that the change in refractive index per degree rise in temperature is $-0.00036$ for a methylated xylose, the indices quoted are regarded as equivalent. In an estimation of methoxy groups by Zeisel's method, 0.1151 g gave 0.5208 g of silver iodide. Found OMe 59.7 where theory requires OMe 60.2 per cent.

In purified chloroform, $[\alpha]_D^{20}$ was $+121.2°$ (0.7273 g having an observed rotation of 7. 06° in 25 ml of solution) and in water $+112.2°$, values not appreciably changed by redistillation. After one redistillation, $n_D^{22.8}$ of the fifteenth drop was 1.4398, that of the last, at 23°, 1.4397, while 0.4710 g in 25 ml of chloroform solution had $\alpha = 4.59°$ and $[\alpha]_D^{20} = +121.7°$. Trimethyl $\alpha$-methylxyloside, therefore, has a specific rotation in chloroform of the order $+121.5°$. Similar measurements were made in water and in methyl alcohol.

$[\alpha]_D^{20}$ in water $= +112.7°$ (0.2942 g in 25 ml gave a rotation of 8.61° in a 2 dm tube).

$[\alpha]_D^{19}$ in $CH_3OH = +122.2$ (0.3791 g in 25 ml gave a rotation of $+3.70°$ in a 2 dm tube).

### 3. PREPARATION AND PROPERTIES OF TRIMETHYL α-XYLOSE FROM EACH OF THE THREE SUBSTANCES DESCRIBED UNDER 1 AND 2

(*a*) Pure trimethyl $\beta$-methylxyloside, 6.29 g, dissolved in 120 ml of 4 per cent hydrochloric acid, was maintained at a temperature of 85° C. The initial specific rotation of $-73.1°$ increased, during 75 minutes, to a nearly constant value of $+14.8°$ whereupon the solution was neutralized with barium carbonate and filtered. In this preparation trimethyl xylose was isolated by a tedious extraction of the filtrate with large volumes of chloroform and by evaporation of the anhydrous extract to a sirup. A partial crystallization quickly occurred, and the crystals were recrystallized from ice-cold ethyl acetate. Yield, 2.3 g; m. p., 91°–92°; $[\alpha]_D^{20} + 54.3°$ in chloroform. The original mother liquors, on standing at 0° for two days, deposited a further 1.2 g; m. p., 88°–91°; $[\alpha]_D^{20} + 53.8°$ in chloroform, while further quantities were obtained from the residual sirup on longer standing. Eventually the yield of crystalline trimethyl xylose reached 85 per cent of the theoretical. After two recrystallizations from ether, 0.5309 g in 25 ml of the chloroform solution had a rotation of 2.40°, or $[\alpha]_D^{20} = +56.4°$. The third and fourth crystallizations raised this figure to $[\alpha]_D^{20} = +58.0°$ (0.3223 g in 25 ml of chloroform had $\alpha = 1.50°$), a value which was not obtained when climatic conditions were hotter and more humid.

*Mutarotation in water.*—0.2988 g, dissolved in 25.10 ml had $\alpha = 0.55°$ twenty minutes after solution while the equilibrium rotation of

+0.43° was reached in three hours. The corresponding specific rotations are $[\alpha]_D^{20} = +23°$ and $[\alpha]_D^{20} = +17.9°$, respectively.

(*b*) Recrystallized trimethyl β-methylxyloside, $[\alpha]_D^{20}$ in $CHCl_3 = -59.6$, prepared by the direct methylation of xylose by methyl sulphate and alkali, was hydrolyzed under conditions similar to those detailed above. The specific rotation of the solution became practically constant at +16.2°, and as this corresponded to an almost quantitative yield of trimethyl xylose, the acid was neutralized with an excess of lead carbonate. Filtration removed the greater part of the inorganic salts from the well-cooled liquor and evaporation of the filtrate under diminished pressure left a dry residue from which hot ether readily extracted trimethyl xylose. The sirupy product crystallized in stages when kept in the ice box for 10 days, the final yield of crystalline material being 75 per cent of theory. Recrystallized from ether, m. p. was 90° and the mixed m. p. with the specimen from (*a*) was 91° to 92°. Mutarotation occurred in chloroform; 0.4477 g of the trimethyl xylose, in 25 ml of solvent, gave a reading of 1.99°, or $[\alpha]_D^{20} = +55.8°$, falling in 90 minutes to the constant value of 0.86° or $[\alpha]_D^{20} = +24.2°$, not changed by catalysis with a trace of acid. In water the first observation was made one minute after solution, 0.2392 g in 25 ml having $\alpha = 1.16°$, decreasing to the constant value of 0.34° in one hour. The corresponding specific rotations were $[\alpha]_D^{20} = +62.8°$ and final $[\alpha]_D^{20} + 17.6°$.

(*c*) Liquid trimethyl α-methylxyloside (2.48 g) was dissolved in 4 per cent hydrochloric acid and the volume increased to 50 ml by addition of the same reagent. When maintained at 85°, the specific rotation of the solution decreased from +109° to +19.4° in the course of 270 minutes. The yield of trimethyl xylose, isolated as in (*b*), was 80 per cent of that theoretically possible; m. p. 91° to 92°, mixed m. p. with the two specimens obtained above, 91° to 92° in each case.

*Mutarotation in water.*—0.2810 g in 25 ml of aqueous solution was read in a 2 dm tube at 22° C.

| Minutes | 0.5 | 2 | 3 | 4 | 5 | 8 | 13 | 21 | 24 | 34 |
|---|---|---|---|---|---|---|---|---|---|---|
| $[\alpha]_D^{20}$ | 62.1 | 57.4 | 55.2 | 52.4 | 50.8 | 43.9 | 35.5 | 28.4 | 27.1 | 22.8 |
| Minutes | 48 | 88 | 123 | 180 | | | | | | |
| $[\alpha]_D^{20}$ | 19.4 | 18.5 | 17.7 | 17.7 | | | | | | |

Extrapolation from these data gave a value of +64° for the initial specific rotation of trimethyl α xylose in water, while in a duplicate experiment +65° was the figure obtained.

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WASHINGTON, December, 19, 1928.

# THE MUTUAL INDUCTANCE OF TWO PARALLEL CIRCLES

By Chester Snow

ABSTRACT

It is shown that the mutual inductance of two parallel circles is identical with the electrostatic potential due to a fairly simple surface charge upon an annular surface. This potential is formulated as a complex line integral, and by deformation of the path its harmonic expansion is obtained for all possible cases.

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## I. INTRODUCTION

If a fixed circle of radius $a_1$ lies in the $y$, $z$ plane with center at the origin, the position of a second circle parallel to the first whose radius is $a_2$ may be specified either by the cylindrical coordinates $x$, $\rho$, $\phi$, or the spherical coordinates $r$, $\theta$, $\phi$, of its center $P$. It is an interesting fact that the mutual inductance $M(x, \rho)$ (independent of $\phi$) of the two circles satisfies Laplace's equation in the coordinates of $P$, and $M$ may, therefore, be identified with the electrostatic potential due to a certain distribution of electricity. This was pointed out by Butterworth[1] who did not determine the equivalent distribution of electrostatic charge, but who made use of the fact that $\nabla^2 M = 0$, to find the expansion for $M$, in zonal harmonics for the case where $r > a_1 + a_2$. For such a type of expansion there are, in fact, three cases to be considered according as the second circle, when brought into the plane of the first by keeping the distance between centers fixed, intersects the first circle or falls wholly within or without it. Butterworth's formula for the case where $r > a_1 + a_2$ is complete, and his treatment of the case of equal circles when $r < 2a$ would be so except for the fact that he obtained only the first four terms of the expansion. The same objection applies to his treatment of the case of "unequal circles close together." Moreover, the expression for the

[1] Butterworth, Phil. Mag., 31, p. 443; 1916.

mutual inductance of coaxial circles upon which Butterworth founds his solution in this case is an expansion of the type

$$M(x, o)=\sum_{k=0}^{\infty}\left[1+\frac{x^2}{(a_1-a_2)^2}\right]^k\left[A_k \log\left(1+\frac{x^2}{(a_1-a_2)^2}\right)+B_k\right]$$

and it is easy to show that such an expression can not be valid for the entire range $a_1-a_2<x<a_1+a_2$, and indeed, if $\frac{a_2}{a_1}<3-2\sqrt{2}=0.172$, it can never be valid, but there must be negative powers of $1+\frac{x^2}{(a_1-a_2)^2}$ in the expansion.

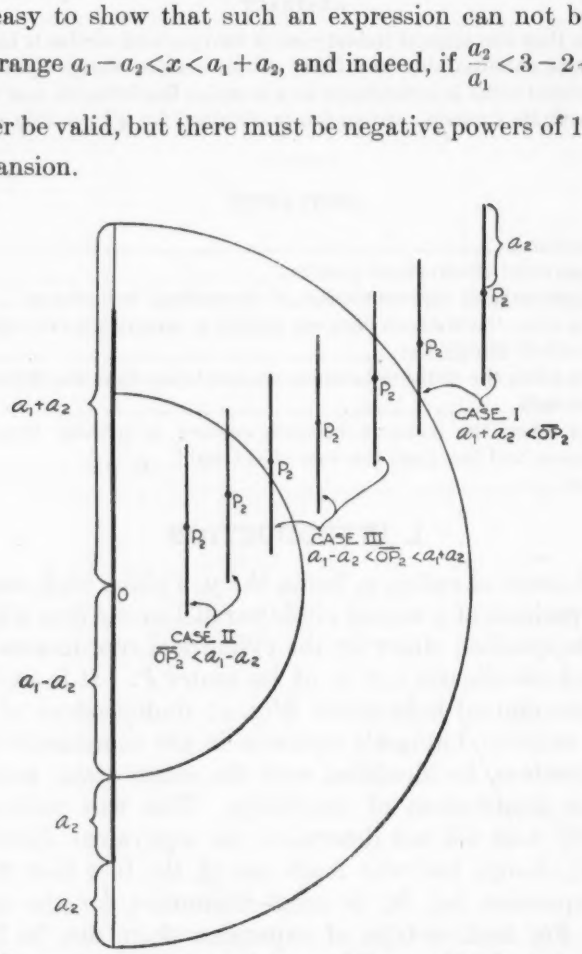


FIGURE 1.—*Showing various relative positions of the small circle (whose center is $P_2$) with respect to larger one whose center is at 0*

Butterworth tested his solution for this case by showing that it would (if completely carried out) give the correct solution for equal circles and also in some cases not clearly defined when the circles were unequal, but coplanar. That these tests are not crucial may be seen from the presence of a series in the solution about to be given of the type $\sum_{k=1}^{\infty} A_k\left(\frac{a_1-a_2}{r}\right)^{2k} P_{2k-1}(\cos\theta)$ which vanishes when the

circles are equal ($a_1=a_2$), and also when they are unequal but coplanar, thus satisfying both the above tests without vanishing in general.

In the following it is shown that the mutual inductance $M(r,\mu)$ of the two circles, the center of the smaller being at the point $r$, $\mu(=\cos\theta)$, is identical with the electrostatic potential due to a symmetrical surface charge of electricity, of total amount zero, upon the annulus in the plane $x=0$ whose boundaries are the two concentric circles $\rho=a_1+a_2$ and $\rho=a_1-a_2$ where $\rho^2=y^2+z^2$. The surface density $\sigma$ of this distribution is found to be a simple function of $\rho$, equation (10), and its Newtonian potential is expressed as a complex line integral. By deforming the path of this integral the residues of the poles of the integrand give the zonal harmonic development of this potential for the various cases. The three possible cases are shown in Figure 1.

## II. THE EQUIVALENT ELECTROSTATIC PROBLEM

It has been shown by the writer [2] that the mutual inductance of any two parallel circles is given by the equation

$$M=4\pi^2 a_1 a_2\int_0^\infty e^{-sx}J_0(\rho s)J_1(a_1 s)J_1(a_2 s)ds \tag{1}$$

If we make use of Gegenbauer's formula [3]

$$J_1(a_1 s)J_1(a_2 s)=\frac{a_1 a_2 s}{\pi}\int_0^\pi \frac{J_1(s\rho_1)}{\rho_1}\sin^2\psi d\psi \tag{2}$$

where $\rho_1$ is defined by

$$\rho_1^2=a_1^2-2a_1a_2\cos\psi+a_2^2 \tag{3}$$

the expression (1) becomes

$$\begin{aligned}M&=4\pi a_1^2a_2^2\int_0^\infty se^{-sx}J_0(\rho s)ds\int_0^\pi\frac{J_1(s\rho)}{\rho_1}\sin^2\psi d\psi\\&=4\pi a_1^2a_2^2\int_0^\pi\frac{\sin^2\psi d\psi}{\rho_1}\int_0^\infty se^{-sx}J_0(\rho s)J_1(\rho_1 s)ds\\&=-4\pi a_1^2a_2^2\int_0^\pi\frac{\sin^2\psi d\psi}{\rho_1}\frac{\partial}{\partial\rho_1}\int_0^\infty e^{-sx}J_0(\rho s)J_0(\rho_1 s)ds\end{aligned}$$

If we use $\rho_1$ instead of $\psi$ for the variable of integration, then by equation (3)

$$-4\pi a_1^2a_2^2\frac{\sin^2\psi d\psi}{\rho_1}=-2\pi\sqrt{[(a_1+a_2)^2-\rho_1^2]\,[\rho_1^2-(a_1-a_2)^2]}\,d\rho_1$$

and

$$M=-2\pi\int_{a_1-a_2}^{a_1+a_2}\sqrt{[(a_1+a_2)^2-\rho_1^2][\rho_1^2-(a_1-a_2)^2]}d\rho_1\frac{\partial}{\partial\rho_1}\int_0^\infty e^{-sx}J_0(\rho_1 s)J_0(\rho_1 s)ds$$

[2] B. S. Jour. Research, **1**, p. 537; 1928.
[3] N. Nielsen, Handbuch der Theorie der Cylinderfunktionen, p. 183.

Integrating by parts (with respect to $\rho_1$) gives

$$M=2\pi\int_{a_1-a_2}^{a_1+a_2}d\rho_1\left(\frac{d}{d\rho_1}\sqrt{[(a_1+a_2)^2-\rho_1^2][\rho_1^2-(a_1-a_2)^2]}\right)\cdot$$

$$\cdot\int_0^\infty e^{-sx}J_0(\rho s)J_0(\rho_1 s)ds \tag{4}$$

Now from the well-known integral formula

$$\int_0^\infty e^{-sx}J_0(sR)\,ds=\frac{1}{\sqrt{x^2+R^2}} \tag{5}$$

If we now let

$$R^2=\rho^2-2\rho\rho_1\cos(\phi-\phi_1)+\rho_1^2$$

then by the use of Neumann's addition formula

$$J_0(sR)=J_0(\rho s)J_0(\rho_1 s)+2\sum_{n=1}^{\infty}J_n(\rho s)J_n(\rho_1 s)\cos n(\phi-\phi_1) \tag{6}$$

it follows that

$$\frac{1}{\sqrt{x^2+\rho^2-2\rho\rho_1\cos(\phi-\phi_1)+\rho_1^2}}= \tag{7}$$

$$=\int_0^\infty e^{-xs}J_0(\rho s)J_0(\rho_1 s)\,ds+2\sum_{n=1}^{\infty}\cos n(\phi-\phi_1)\int_0^\infty e^{-sx}J_n(\rho s)J_n(\rho_1 s)\,ds$$

If this equation be multiplied by $d\phi_1$ and integrated from 0 to $2\pi$, it gives

$$2\pi\int_0^\infty e^{-xs}J_0(\rho s)J_0(\rho_1 s)\,ds=\int_0^{2\pi}\frac{d\phi_1}{\sqrt{x^2+\rho^2-2\rho\rho_1\cos(\phi-\phi_1)+\rho_1^2}} \tag{8}$$

By the aid of this, the expression (4) for $M$ may be written

$$M=\int_{a_1-a_2}^{a_1+a_2}d\rho_1\int_0^{2\pi}\frac{\rho_1\sigma(\rho_1)\,d\phi_1}{\sqrt{x^2+\rho^2-2\rho\rho_1\cos(\phi-\phi_1)+\rho_1^2}} \tag{9}$$

where if

$$a_1-a_2<\rho_1<a_1+a_2$$

$$\sigma(\rho_1)=\frac{1}{\rho_1}\frac{d}{d\rho_1}\sqrt{[(a_1+a_2)^2-\rho_1^2][\rho_1^2-(a_1-a_2)^2]} \tag{10}$$

or

$$\rho_1\sigma(\rho_1)=\frac{2(a_1^2+a_2^2-\rho_1^2)\rho_1}{\sqrt{[(a_1+a_2)^2-\rho_1^2][\rho_1^2-(a_1-a_2)^2]}} \tag{11}$$

The radical in the integrand of the double integral (9) represents the distance from the point of integration $P_1$, whose cylindrical coordinates are $x=0$, $\rho=\rho_1$, $\phi=\phi_1$, to the general point $P(x, \rho, \phi)$, and the

integration extends over an annulus in the $y, z$ plane which is bounded by the concentric circles $\rho_1 = a_1 - a_2$ and $\rho_1 = a_1 + a_2$. Since $\sigma$ is a function of $\rho_1$ only (not involving $\phi_1$), the integral is independent of $\phi$, and we recognize in this integral the Newtonian potential at $P$ due to a symmetrical electrostatic charge on this annulus whose surface density $\sigma$ is the function of $\rho_1$ defined by equation (10). Although this density becomes infinite at the inner and outer edges of the annulus, it does so in such a manner that its Newtonian potential is everywhere finite and continuous. This latter vanishes at infinity to the order of $\frac{1}{r^3}$, since the total charge $2\pi \int_{a_1+a_2}^{a_1+a_2} \rho_1 \sigma d\rho_1$ is zero. This establishes the identity of the mutual inductance of the circles and the electrostatic potential. When the point $P$ comes on to the charged annulus, this corresponds to the two circles becoming coplanar and intersecting.

## III. A COMPLEX-INTEGRAL REPRESENTATION OF THE MUTUAL INDUCTANCE

If we multiply together the binominal series for $\left[1-\left(\frac{\rho_1}{a_1+a_2}\right)^2\right]^{-\frac{1}{2}}$ and for $\left[1-\left(\frac{a_1-a_2}{\rho_1}\right)^2\right]^{-\frac{1}{2}}$ and then rearrange the resulting double series in ascending and descending powers of $\rho_1$, we find an expansion for the function $\rho_1\sigma(\rho_1)$ given by equation (10). Since $\rho_1$ is a value of $r$ when $\theta = \frac{\pi}{2}$—that is, $\mu = \cos\theta = 0$—we may replace it by $r$, and the expansion is

$$-\left[\frac{\partial M(r,\mu)}{\partial \mu}\right]_{\mu=+0} = 2\pi r\sigma(r) = (a_1+a_2)\sum_{k=-\infty}^{k=+\infty}\left(k+\frac{1}{2}\right)b_k(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2k}$$

$$\text{if } a_1 - a_2 < r < a_1 + a_2 \tag{12}$$

$$= o \text{ if } r < a_1 - a_2 \text{ or if } r > a_1 + a_2$$

where for general values of $\nu$, the function of $\nu$ and $\lambda$, $b_\nu(\lambda)$ is defined by

$$b_\nu(\lambda) \equiv \frac{\Gamma\left(-\frac{1}{2}\right)\Gamma\left(\nu-\frac{1}{2}\right)}{\Gamma(\nu+1)} F\left(-\frac{1}{2}, \nu-\frac{1}{2}, \nu+1, \lambda\right)$$

$$= \sum_{s=0}^{\infty} \frac{\Gamma\left(s-\frac{1}{2}\right)\Gamma\left(s+\nu-\frac{1}{2}\right)}{\Gamma(s+1)\Gamma(s+\nu+1)}\lambda^s \tag{13}$$

$$\lambda \equiv \left(\frac{a_1-a_2}{a_1+a_2}\right)^2$$

where $F$ denotes the hypergeometric function. It follows from this definition that when (and only when) $\nu = k =$ an integer (positive, negative or zero), $b_\nu$ satisfies the following equation:

$$b_k(\lambda) - \lambda^{-k} b_{-k}(\lambda) = 0 \tag{14}$$

It is evident from the definition (13) of $b_\nu(\lambda)$ as a function of the complex variable $\nu$ (with $\lambda$ fixed and less than one) that $b_\nu$ is analytic at all finite points of the $\nu$ plane except for the simple poles at $\nu = \frac{1}{2} - k$ where $k = 0, 1, 2, 3 \cdots$ Similarly, the poles of $b_{-\nu}$ are at $\nu = k - \frac{1}{2}$. Consequently all the poles of $b_\nu$ and $b_{-\nu}$ are the totality of zeros of the function $\cos \nu\pi$ and, therefore $b_\nu \cos \nu\pi$ and $b_{-\nu} \cos \nu\pi$ have no poles and $\cos \nu\pi \, (b_\nu - \lambda^{-\nu} b_{-\nu})$ has no poles but vanishes whenever $\nu$ is a real integer; that is, for all points at which $\sin \nu\pi$ vanishes. Consequently the function $g_\nu(\lambda)$ defined by

$$g_\nu(\lambda) \equiv -\pi \cot \nu\pi (b_\nu(\lambda) - \lambda^{-\nu} b_{-\nu}(\lambda)) \tag{15}$$

is analytic at all finite points in the $\nu$ plane. It does not vanish at the poles of $b_\nu$ and $b_{-\nu}$, nor at the poles of $\cot \nu\pi$. This function so defined satisfies identically the equation

$$g_\nu(\lambda) - \lambda^{-\nu} g_{-\nu}(\lambda) \equiv 0 \tag{16}$$

for all values of $\nu$, whereas the corresponding equation (14) for $b_\nu$ only holds when $\nu$ is integral.

We find a simpler expression for the function $g_\nu(\lambda)$ by applying Gauss' transformation to the hypergeometric function $F\left(\frac{3}{2}, \nu + \frac{3}{2}, 3, 1 - \lambda\right)$, which gives

$$g_\nu(\lambda) = \frac{\pi^2}{2} (1-\lambda)^2 F\left(\frac{3}{2}, \nu + \frac{3}{2}, 3, 1 - \lambda\right) \tag{17}$$

$$= 8\pi^2 \frac{a_2^2}{a_1^2} \left(\frac{a_1 + a_2}{a_1}\right)^{2\nu - 1} F\left(\nu + \frac{1}{2}, \nu + \frac{3}{2}, 2, \frac{a_2^2}{a_1^2}\right)$$

(by Kummer's transformation).

This also shows that $g_\nu(\lambda)$ has no poles in the complex plane, and that (16) is an identity. By using Stirling's asymptotic expansion for the Gamma function in the series (13) which defines $b_\nu(\lambda)$, it may be shown that if $\nu$ is not a negative real,

$$\lim_{|\nu| \to \infty} b_\nu(\lambda) = 0 \text{ (to the order } \nu^{-\frac{3}{2}})$$

Hence, if $\nu$ is not a positive real,

$$\lim_{|\nu|\to\infty} b_{-\nu}(\lambda) = 0 \text{ (to the order } \nu^{-\frac{3}{2}})$$

Also since

$$\lim_{|\nu|\to\infty} \cot \nu\pi = 1$$

if $\nu$ is not real, it is evident from equation (15) that

$$\lim_{|\nu|\to\infty} \cot \nu\pi b_{\nu}(\lambda) = 0 \text{ and } \lim_{|\nu|\to\infty} \cot \nu\pi b_{-\nu}(\lambda) = 0$$

if $\nu$ is not real.

In the equation (15) $\lambda^{-\nu}$ must be understood to represent the principal branch $e^{-\nu \log \lambda}$ where $\log \lambda$ is the real (natural) logarithm of $\lambda$. It is evident from (15) that if $\nu$ is not a pure real, $\lim_{|\nu|\to\infty} g_{\nu}(\lambda) = 0$ if the real part of $\nu$ is negative, but $g_{\nu}(\lambda)$ becomes infinite with infinite $|\nu|$ if the real part of $\nu$ is positive. However, in this case $\lim_{|\nu|\to\infty} c^{\nu} g_{\nu}(\lambda) = 0$ if the real part of $\nu$ is positive and $c$ is a positive real quantity less than $\lambda$.

If we desire a representation of the discontinuous function of $r$, namely, $\left[\frac{\partial M(r,\mu)}{\partial \mu}\right]_{\mu=0}$ as given by equation (12) in the form of an integral $\int f(\nu) r^{2\nu} d\nu$ along some path in the complex $\nu$-plane, the equation (16) suggests that the poles of $f(\nu)$ must be the real integers, and it gives the value of the residues. No great intuition is then required to find that $\left(\frac{\partial M}{\partial \mu}\right)_{\mu=0}$ may be represented by the following path integral in the $\nu$-plane

$$\left[\frac{\partial M(r,\mu)}{\partial \mu}\right]_{\mu=+0} = \frac{a_1+a_2}{2\pi i}\int_{i\infty}^{-i\infty}\left(\nu+\frac{1}{2}\right) g_{\nu}(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu} d\nu$$

$$= \frac{\pi(a_1+a_2)(1-\lambda)^2}{4i}\int_{i\infty}^{-i\infty}\left(\nu+\frac{1}{2}\right) F\left(\frac{3}{2}, \nu+\frac{3}{2}, 3, 1-\lambda\right)\left(\frac{r}{a_1+a_2}\right)^{2\nu} d\nu \quad (18)$$

$$= \frac{(a_1+a_2)}{2i}\int_{i\infty}^{-i\infty}\left(\nu+\frac{1}{2}\right)\cot \nu\pi\left\{b_{-\nu}(\lambda)\left(\frac{r}{a_1-a_2}\right)^{2\nu} - b_{\nu}(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu}\right\} d\nu$$

where the path of the integral is down the entire imaginary axis of $\nu$ from $+i\infty$ to $-i\infty$ except that it crosses the real axis to the left of the origin and to the right of the point $\nu = -1$. In this integral $\left(\frac{r}{a_1+a_2}\right)^{2\nu} = e^{\nu \log\left(\frac{r}{a_1+a_2}\right)^2}$ where $\log\left(\frac{r}{a_1+a_2}\right)^2$ denotes the real (natural) logarithm.

To verify this induction consider first the case where $r > a_1 + a_2$ so that $\lim_{|\nu|\to\infty}\left(\frac{r}{a_1+a_2}\right)^{2\nu} = 0$ if the real part of $\nu$ is negative, in which case

we have seen $g_\nu$ also vanishes. In this case we may make the path of the integral in equation (18) into a closed contour by adding to it a semicircular path of infinite radius, center at origin and lying to the left of the imaginary axis. There are no poles of the integrand in this contour, and on shrinking it to the vanishing point we conclude from equation (18) that

$$\left(\frac{\partial M}{\partial \mu}\right)_{\mu=+0} = 0 \text{ if } r > a_1 + a_2$$

Consider next the case where $r < a_1 - a_2$, and, hence, $0 < \frac{r}{a_1 + a_2} < \lambda$.

In this case we have just seen that $\underset{|\nu| \to \infty}{\text{limit}} \left(\frac{r}{a_1 + a_2}\right)^{2\nu} g_\nu(\lambda) = 0$ and also $\left(\nu + \frac{1}{2}\right)\left(\frac{r}{a_1 + a_2}\right)^{2\nu} g_\nu(\lambda) = 0$ if the real part of $\nu$ is positive.

In this case we add a zero quantity to the integral in equation (18) by closing its path with an infinite semicircle lying to the right of the imaginary axis. As this contour contains no poles of the integrand, we deduce from equation (18) that

$$\left(\frac{\partial M}{\partial \mu}\right)_{\mu=+0} = 0 \text{ if } r < a_1 - a_2$$

Finally, consider the case where $a_1 - a_2 < r < a_1 + a_2$, so that $\frac{r}{a_1 - a_2} > 1$ and $\frac{r}{a_1 + a_2} < 1$.

Breaking up the integral as indicated in the last form of equation (18), the path of the integral

$$\frac{a_1 + a_2}{2i} \int \left(\nu + \frac{1}{2}\right) \cot \nu\pi b_{-\nu}(\lambda) \left(\frac{r}{a_1 - a_2}\right)^{2\nu} d\nu$$

may be similarly made into a closed contour encircling (in the negative direction) that part of the real axis to the left of the point $\nu = -1$ (inclusive), where all the poles of the integrand are the negative real integers. Since the integral $\int \cot \nu \, \pi \, d\nu$ taken in the negative direction around an infinitesimal loop encircling any one of these integers is just $-2i$, this integral reduces to

$$-(a_1 + a_2) \sum_{k=-\infty}^{-1} \left(k + \frac{1}{2}\right) b_{-k}(\lambda) \left(\frac{r}{a_1 - a_2}\right)^{2k}$$

$$= -(a_1 + a_2) \sum_{k=-\infty}^{-1} \left(k + \frac{1}{2}\right) b_k(\lambda) \left(\frac{r}{a_1 + a_2}\right)^{2k}$$

by equation (14). Similarly the path of the integral

$$-\frac{a_1+a_2}{2i}\int\left(\nu+\frac{1}{2}\right)\cot\nu\pi b_\nu(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu}d\nu$$

may be made a closed contour encircling (in the positive direction) all the positive real axis (including the origin). The residues from the poles of $\cot\nu\pi$, at $k=0, 1, 2, 3 \ldots$ gives for this integral

$$-(a_1+a_2)\sum_{k=0}^{\infty}\left(k+\frac{1}{2}\right)b_k(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2k}$$

These two series give the development of $\left(\frac{\partial M}{\partial\mu}\right)_{\mu=0}=-2\pi\rho\sigma$ by equation (12).

Consequently the problem to be solved is to find a function $M(r, \mu)$ which, in the range $0 \leqq \mu \leqq 1$, $0 \leqq r \leqq +\infty$, satisfies Laplace's equation, vanishes when $r\to\infty$ and also satisfies the boundary condition in the form given by equation (18). Since

$$\nabla^2(r^{2\nu}P_{2\nu}(\mu))=0 \text{ where } P_{2\nu}(\mu)=F\left(2\nu+1, -2\nu, 1, \frac{1-\mu}{2}\right) \tag{19}$$

the solution of the problem is

$$M(r,\mu)=\frac{a_1+a_2}{2\pi i}\int_{i\infty}^{-i\infty}\left(\nu+\frac{1}{2}\right)g_\nu(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu}\frac{P_{2\nu}(\mu)}{P'_{2\nu}(0)}d\nu \tag{20}$$

or since

$$P'_{2\nu}(0)=-\frac{2\sqrt{\pi}}{\Gamma\left(\nu+\frac{1}{2}\right)\Gamma(-\nu)} \tag{21}$$

$$\begin{aligned}M(r,\mu)&=-\frac{a_1+a_2}{4\pi^{3/2}i}\int_{i\infty}^{-i\infty}\Gamma(-\nu)\Gamma\left(\nu+\frac{3}{2}\right)g_\nu(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu}P_{2\nu}(\mu)d\nu\\&=\frac{a_1+a_2}{4i\sqrt{\pi}}\int_{i\infty}^{-i\infty}\Gamma(-\nu)\Gamma\left(\nu+\frac{3}{2}\right)\cot\nu\pi\,P_{2\nu}(\mu)\left\{b_\nu(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu}\right.\\&\left.-b_{-\nu}(\lambda)\left(\frac{r}{a_1-a_2}\right)^{2\nu}\right\}d\nu\end{aligned} \tag{22}$$

If $0<\mu\leqq 1$, this integral defines a harmonic function, continuous with its derivatives, satisfying the boundary condition (18) at $\mu=+0$, and (as will be shown presently) one which vanishes when $r\to\infty$. A comparison of the last form of the integral (22) with the last form of (18) shows that we have introduced an additional factor

$$\Gamma(-\nu)\Gamma\left(\nu+\frac{1}{2}\right)P_{2\nu}(\mu)$$

into the integrand, which introduces new poles, all on the real axis. Since $P_{2\nu}(\mu)$ is defined as $F\left(2\nu+1, -2\nu, 1, \frac{1-\mu}{2}\right)$, it may be shown by Stirling's expansion that when $\nu$ is not real this new factor, when it becomes infinite with infinite value of $|\nu|$, does so in such a manner that its product by $\left(\frac{r}{a_1+a_2}\right)^{\nu}$ will vanish if the real part of $\nu$ is positive, for example, when $r<a_1+a_2$. Consequently it is possible in each case to make the same deformation of path for the integral (22) as was made in the corresponding case in discussion of the integral (18). We proceed to do this and to obtain the zonal harmonic expansions for $M$ in the three cases that arise.

## IV. FORMULA WHEN THE DISTANCE BETWEEN CENTERS IS GREATER THAN THE SUM OF THE RADII OF THE CIRCLES

If $\left(\frac{r}{a_1+a_2}\right)>1$, then $\left(\frac{r}{a_1+a_2}\right)^{2\nu}\to 0$ when $|\nu|\to\infty$, the real part of $\nu$ being negative. Hence, in this case we may swing the path of the integral in equation (22) to the left until it closes down upon the negative real axis to the left of and including the point $\nu=-1$. Here we meet the only poles of the integrand—those which make the factor $\Gamma\left(\nu+\frac{3}{2}\right)$ become infinite—namely, $\nu=-k-\frac{3}{2}$, where $k=0, 1, 2, 3, \cdots$ Since $\frac{1}{2\mu i}\int\Gamma\left(\nu+\frac{3}{2}\right)d\nu$ taken positively around an infinitesimal loop inclosing the point $\nu_k=-k-\frac{3}{2}$ is just $\frac{(-1)^k}{\Gamma(k+1)}$ if $k=0, 1, 2, 3, \cdots$ the integral (22) reduces to

$$M=\frac{a_1+a_2}{2\sqrt{\pi}}\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}g_{-k-\frac{1}{2}}(\lambda)\left(\frac{a_1+a_2}{r}\right)^{2k+3}P_{-2k-3}(\mu)$$

But since $P_\nu(\mu)=P_{-\nu-1}(\mu)$ for all values of $\nu$

$$P_{-2k-3}(\mu)=P_{2k+2}(\mu)$$

Also by the identical property of $g_\nu(\lambda)$ expressed by equation (16)

$$g_{-k-\frac{1}{2}}(\lambda)=\lambda^{k+\frac{1}{2}}g_{k+\frac{1}{2}}(\lambda)=\left(\frac{a_1-a_2}{a_1+a_2}\right)^{2k+3}g_{k+\frac{1}{2}}(\lambda)$$

Hence, for this case we find

$$M(r,\mu)=-\frac{a_1+a_2}{2\sqrt{\pi}}\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k)}g_{k+\frac{1}{2}}(\lambda)\left(\frac{a_1-a_2}{r}\right)^{2k+1}P_{2k}(\mu) \quad (23a)$$

Inserting the two forms of $g_{k+\frac{1}{2}}(\lambda)$ given by equation (17) gives

$$M(r,\mu)=-\frac{(a_1+a_2)\pi^{3/2}(1-\lambda)^2}{4}\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k)}\cdot$$

$$\cdot\left(\frac{a_1-a_2}{r}\right)^{2k+1}P_{2k}(\mu)\,F\left(\frac{3}{2},k+2,3,1-\lambda\right) \tag{23b}$$

$$=-\frac{(a_1+a_2)\pi^{3/2}(1-\lambda)^2}{4}\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k)}\cdot$$

$$\cdot\left(\frac{a_1+a_2}{r}\right)^{2k+1}P_{2k}(\mu)F\left(\frac{3}{2},1-k,3,1-\lambda\right)$$

by the use of equation (16). Also

$$M(r,\mu)=-\frac{4\pi^{3/2}a_2^{\,2}}{a_1}\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k)}\left(\frac{a_1^{\,2}-a_2^{\,2}}{a_1 r}\right)^{2k+1}\cdot$$

$$\cdot P_{2k}(\mu)\,F\left(k+1,k+2,2,\frac{a_2^{\,2}}{a_1^{\,2}}\right) \tag{23c}$$

$$=-\frac{4\pi^{3/2}a_2^{\,2}}{a_1}\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k)}\left(\frac{a_1}{r}\right)^{2k+1}P_{2k}(\mu)F\left(-k,1-k,2,\frac{a_2^{\,2}}{a_1^{\,2}}\right)$$

This is Butterworth's formula. The following form may also be of interest:

$$M(r,\mu)=-\frac{4\pi^{3/2}a_1^{\,2}a_2^{\,2}}{(a_1^{\,2}+a_2^{\,2})^{\frac{3}{2}}}\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k)}\left(\frac{a_1^{\,2}+a_2^{\,2}}{r^2}\right)^{k+\frac{1}{2}}\cdot \tag{23d}$$

$$\cdot P_{2k}(\mu)\,F\left(1-\frac{k}{2},\frac{1}{2}-\frac{k}{2},2,\left(\frac{2a_1a_2}{a_1^{\,2}+a_2^{\,2}}\right)^2\right)$$

When the circles are nearly equal it is better to have coefficients which are expressible in ascending powers of $\lambda\left[=\left(\frac{a_1-a_2}{a_1+a_2}\right)^2\right]$. For this purpose we go back to (23a) and evaluate $g_{k+\frac{1}{2}}(\lambda)$ from its definition (15). It takes the indeterminate form $0.\infty$ but is easily evaluated, and we find

$$g_{k+\frac{1}{2}}(\lambda)=\operatorname*{limit}_{\nu\to k+\frac{1}{2}}\left[\lambda^{-\nu}b_{-\nu}(\lambda)\,\pi\cot\nu\pi\right]$$

$$=\frac{2\pi^{\frac{1}{2}}(1-\lambda)^2}{\lambda^{k+\frac{1}{2}}}\frac{\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k+2)}F\left(\frac{3}{2},1-k,\frac{1}{2}-k,\lambda\right)$$

so that (23a) takes the more convenient form

$$M(r,\mu)=-\pi(a_1+a_2)(1-\lambda)^2\sum_{k=1}^{\infty}(-1)^k\cdot$$

$$\cdot\frac{\Gamma\left(k+\frac{1}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k)\,\Gamma(k+2)}\left(\frac{a_1+a_2}{r}\right)^{2k+1}P_{2k}(\mu)F\left(\frac{3}{2},1-k,\frac{1}{2}-k,\lambda\right) \tag{23e}$$

Also Gauss' transformation changes (23d) to the form

$$M(r,\mu)=-\frac{2\pi(a_1^2+a_2^2)}{r}\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{1}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k)\,\Gamma(k+2)}\cdot$$

$$\cdot\left(\frac{2(a_1^2+a_2^2)}{r^2}\right)^k P_{2k}(\mu)F\left(-\frac{k}{2},-\frac{k+1}{2},\frac{1}{2}-k,\left(\frac{a_1^2-a_2^2}{a_1^2+a_2^2}\right)^2\right) \tag{23f}$$

If the circles are equal, $a_1=a_2=a$ and $\lambda=\left(\frac{a_1-a_2}{a_1+a_2}\right)^2=0$, and these reduce to

$$M(r,\mu)=-2\pi a\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{1}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(k)\Gamma(k+2)}\left(\frac{2a}{r}\right)^{2k+1}P_{2k}(\mu)\ \text{if}\ r>2a \tag{24}$$

## V. FORMULA WHEN THE DISTANCE BETWEEN CENTERS IS LESS THAN THE DIFFERENCE OF THE RADII

In this case $\frac{r^2}{(a_1+a_2)^2}<\lambda$, hence $\underset{|\nu|\to\infty}{\text{limit}}\left(\frac{r}{a_1+a_2}\right)^{2\nu}g_\nu(\lambda)=0$ if the real part of $\nu$ is positive. This vanishes exponentially, so that if $\nu$ is not real, the additional factors $\Gamma(-\nu)\Gamma\left(\nu+\frac{3}{2}\right)$ do not prevent the integrand from vanishing. The path of the integral in (22) may, therefore, be shrunk to one encircling in a positive direction the positive real axis, including the origin. Here the poles of $\Gamma(-\nu)$ at $\nu=k=0,1,2,3\cdots\cdots$ give the residues

$$\frac{1}{2\pi i}\int\Gamma(-\nu)\,d\nu=-\frac{(-1)^k}{\Gamma(k+1)}$$

so that the integral (22) in this case reduces to

$$M(r,\mu)=\frac{a_1+a_2}{2\sqrt{\pi}}\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}g_k(\lambda)\left(\frac{r}{(a_1+a_2)}\right)^{2k}P_{2k}(\mu) \tag{25a}$$

Or, using the two values of $g_k(\lambda)$ given in equation (17),

$$M(r,\mu)=\frac{(a_1+a_2)\,\pi^{\frac{1}{2}}(1-\lambda)^2}{4}\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}\cdot$$

$$\cdot\left(\frac{r}{(a_1-a_2)}\right)^{2k}P_{2k}(\mu)\,F\left(\frac{3}{2},\frac{3}{2}-k,3,1-\lambda\right) \tag{25b}$$

and

$$M(r,\mu)=4\pi^{\frac{1}{2}}\frac{a_2^2}{a_1}\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1}\cdot$$

$$\cdot\left(\frac{r}{a_1}\right)^{2k}P_{2k}(\mu)\,F\left(k+\frac{1}{2},k+\frac{3}{2},2,\frac{a_2^2}{a_1^2}\right) \tag{25c}$$

There is also the analogue of (23d),

$$M(r,\mu)=\frac{4\pi^{\frac{1}{2}}a_1^2a_2^2}{(a_1^2+a_2^2)^{\frac{3}{2}}}\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}\left(\frac{r^2}{(a_1^2+a_2^2)}\right)^k\cdot$$

$$\cdot P_{2k}(\mu)\,F\left(\frac{k}{2}+\frac{3}{4},\frac{k}{2}+\frac{5}{4},2,\left(\frac{2a_1a_2}{a_1^2+a_2^2}\right)^2\right) \tag{25d}$$

If $\lambda$ is small, the coefficients in (25a) may be computed by the following formula obtained by evaluating the $\lim_{\nu=k} g_\nu(\lambda)$ using (13), (14), and (15). This gives for $k=0, 1, 2, 3\cdots\cdots$

$$g_k(\lambda)=-b_k(\lambda)\log\lambda+\sum_{s=-k}^{-1}(-1)^s\frac{\Gamma(-s)\Gamma\left(s-\frac{1}{2}\right)\Gamma\left(s+k-\frac{1}{2}\right)}{\Gamma(s+k+1)}\lambda^s$$

$$-\sum_{s=0}^{\infty}\lambda^s\frac{\Gamma\left(s-\frac{1}{2}\right)\Gamma\left(s+k-\frac{1}{2}\right)}{\Gamma(s+1)\,\Gamma(s+k+1)}\left[\psi\left(s-\frac{1}{2}\right)+\psi\left(s+k-\frac{1}{2}\right)\right.$$

$$\left.-\psi(s+k+1)-\psi(s+1)\right]$$

where $\psi(z)$ is defined in equation (29) below. The formula (25c) was derived by the writer in an earlier publication (see footnote 2).

## VI. FORMULA WHEN THE DISTANCE BETWEEN CENTERS IS GREATER THAN THE DIFFERENCE AND LESS THAN THE SUM OF THE RADII

Consider first that part of the integral (22) which is

$$-\frac{a_1+a_2}{4i\sqrt{\pi}}\int_{i\infty}^{-i\infty}\Gamma(-\nu)\,\Gamma\left(\nu+\frac{3}{2}\right)\cot\nu\pi\,b_{-\nu}(\lambda)\left(\frac{r}{a_1-a_2}\right)^{2\nu}P_{2\nu}(\mu)\,d\nu$$

Since $\frac{r}{a_1-a_2}$ is now $>1$, the path of this integral may be deformed into a closed contour encircling the negative real axis (in the negative or clockwise direction) and crossing the real axis between the origin and the point $\nu=-1$. The only poles of the integrand are the negative real integers, which are simple poles of $\cot\nu\pi$, for $b_{-\nu}$ has no poles on the negative real axis, and the poles of the factor $\Gamma\left(\nu+\frac{3}{2}\right)$ are the zeros of the factor $\cot\nu\pi$.

Hence, since

$P_{-2k}(\mu)=P_{2k-1}(\mu)$

the above integral is given by

$$-\frac{a_1+a_2}{4i\sqrt{\pi}}\int_{i\infty}^{-i\infty}\Gamma(-\nu)\,\Gamma\left(\nu+\frac{3}{2}\right)\cot\nu\pi\,b_{-\nu}(\lambda)\left(\frac{r}{a_1-a_2}\right)^{2\nu}P_{2\nu}(\mu)\,d\nu=$$

$$=-\frac{a_1+a_2}{2\sqrt{\pi}}\pi\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma(k)}{\Gamma\left(k-\frac{1}{2}\right)}b_k(\lambda)\left(\frac{a_1-a_2}{r}\right)^{2k}P_{2k-1}(\mu) \qquad (26$$

Next consider the remaining integral of (22), which is

$$\frac{a_1+a_2}{4i\sqrt{\pi}}\int_{i\infty}^{-i\infty}\Gamma(-\nu)\Gamma\left(\nu+\frac{3}{2}\right)\cot\nu\pi b_{\nu}(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu}P_{2\nu}(\mu)\,d\nu$$

Since $\frac{r}{a_1+a_2}$ is here $<1$, the path of this integral may be made to encircle (in a positive direction) the entire positive real axis (including the origin). On this side of the plane $b_{\nu}$ has no poles, nor has $\Gamma\left(\nu+\frac{3}{2}\right)$, but the positive integers (and zero) are now second-order poles of the factor $\Gamma(-\nu)\cot\nu\pi$. Since the integral

$$\frac{1}{2\pi i}\int\Gamma(-\nu)\cot\nu\pi f(\nu)d\nu$$

taken around any one of these poles $\nu=k$, where $k=0, 1, 2, 3\cdots$, is just

$$\frac{(-1)^{k+1}}{\pi}\left[\frac{d}{d\nu}\frac{f(\nu)}{\Gamma(\nu+1)}\right]_{\nu=k}$$

it is found that

$$\frac{a_1+a_2}{4i\sqrt{\pi}}\int_{i\infty}^{-i\infty}\Gamma(-\nu)\Gamma\left(\nu+\frac{3}{2}\right)\cot\nu\pi b_\nu(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu}P_{2\nu}(\mu)d\nu= \tag{27}$$

$$=-\frac{a_1+a_2}{2\sqrt{\pi}}\sum_{k=0}^{\infty}(-1)^k\left[\frac{\partial}{\partial\nu}\left\{\frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\Gamma(\nu+1)}b_\nu(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu}P_{2\nu}(\mu)\right\}\right]_{\nu=k}$$

Adding equations (26) and (27), we find for the case where $a_1-a_2<r<a_1+a_2$

$$\frac{2\sqrt{\pi}}{a_1+a_2}M(r,\mu)=-\pi\sum_{k=1}^{\infty}-(1)^k\frac{\Gamma(k)}{\Gamma\left(k-\frac{1}{2}\right)}b_k(\lambda)\left(\frac{a_1-a_2}{r}\right)^{2k}P_{2k-1}(\mu) \tag{28}$$

$$-\sum_{k=0}^{\infty}(-1)^k\left(\frac{\partial}{\partial\nu}\left[\frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\Gamma(\nu+1)}b_\nu(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2\nu}P_{2\nu}(\mu)\right]\right)_{\nu=k}$$

It is evident that each term of this series satifies Laplace's equation, since

$$\nabla^2\frac{\partial}{\partial\nu}f(\nu)r^{2\nu}P_{2\nu}(\mu)=\frac{\partial}{\partial\nu}\nabla^2f(\nu)r^{2\nu}P_{2\nu}(\mu)=0$$

In order to express this series in terms of functions which are suitable for computation, we may introduce the logarithmic derivative of the Gamma function which is usually denoted by $\psi(z)$ and which may be computed by the formula

$$\psi(z)\equiv\frac{\Gamma'(z)}{\Gamma(z)}=-c-\sum_{t=0}^{\infty}\left(\frac{1}{t+z}-\frac{1}{t+1}\right) \tag{29}$$

where $c=0.57721566=$Euler's constant. From the definition (13) of $b_\nu(\lambda)$, one finds

$$\frac{\partial}{\partial k}b_k(\lambda)=\sum_{s=0}^{\infty}\lambda^s\frac{\Gamma\left(s-\frac{1}{2}\right)\Gamma\left(s+k-\frac{1}{2}\right)}{\Gamma(s+1)\Gamma(s+k+1)}\left[\psi\left(s+k-\frac{1}{2}\right)-\psi(s+k+1)\right] \tag{30}$$

Also

$$\frac{\partial}{\partial k}\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}=\left[\psi\left(k+\frac{3}{2}\right)-\psi(k+1)\right]\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}$$

$$=\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}\left[2\log\frac{1}{2}+\frac{2}{2k+1}+\sum_{t=0}^{k-1\geq0}\left(\frac{1}{t+\frac{1}{2}}-\frac{1}{t+1}\right)\right] \tag{31}$$

and

$$\frac{\partial}{\partial k}\left(\frac{r}{a_1+a_2}\right)^{2k}=\left(\frac{r}{a_1+a_2}\right)^{2k}\log\left(\frac{r}{a_1+a_2}\right)^2 \tag{32}$$

These three formulas are valid for any value of $k$. It is well known that if $k$ is a positive integer or zero,

$$\left[\frac{\partial P_\nu(\mu)}{\partial \nu}\right]_{\nu=k}=P_k(\mu)\ \log\left(\frac{\mu+1}{2}\right)+\psi_k(\mu) \tag{33}$$

so that

$$\left[\frac{\partial P_{2\nu}(\mu)}{\partial \nu}\right]_{\nu=k}=2P_{2k}(\mu)\ \log\left(\frac{\mu+1}{2}\right)+2\psi_{2k}(\mu)$$

where $\psi_k(\mu)$ is a polynomial in $\mu$ of degree $k$, which may be computed by either of the following formulas:

$$\left.\begin{aligned}&\psi_k(\mu)=2(-1)^k\sum_{s=0}^{k}(-1)^s\left(\frac{\mu+1}{2}\right)^s\frac{(k+s)!}{s!\,s!\,(k-s)!}[\psi(s+k+1)-\psi(s+1)]\\ &\text{or}\\ &\psi_k(\mu)=2(-1)^k\sum_{s=0}^{k-1}(-1)^s\frac{2s+1}{(s+k+1)\,(s-k)}\,[P_k(\mu)-P_s(\mu)]\end{aligned}\right\} \tag{34}$$

The first form shows that $\psi_o(\mu)=0$ and the second that $\psi_k(1)=0$. (Since $\psi_k(\mu)$ is always written with a subscript, there is no danger of confusing these polynomials with the logarithmic derivative of the gamma function defined by equation (29). The notation in both cases seems well established by usage). Making use of formulas (30) to (33) in the series (28) gives

$$\begin{aligned}\frac{2\sqrt{\pi}}{a_1+a_2}M(r,\mu)=&-\pi\sum_{k=1}^{\infty}(-1)^k\frac{\Gamma(k)}{\Gamma\left(k-\frac{1}{2}\right)}b_k(\lambda)\left(\frac{a_1-a_2}{r}\right)^{2k}P_{2k-1}(\mu)\\ &-\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}b_k(\lambda)\left(\frac{r}{a_1+a_2}\right)^{2k}P_{2k}(\mu)\Bigg[\frac{2}{2k+1}\\ &+\sum_{t=0}^{k-1\geq 0}\left(\frac{1}{t+\frac{1}{2}}-\frac{1}{t+1}\right)+2\log\frac{r(\mu+1)}{4(a_1+a_2)}\Bigg]\\ &-\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}\left(\frac{r}{a_1+a_2}\right)^{2k}\Bigg[P_{2k}(\mu)\frac{\partial b_k(\lambda)}{\partial k}\\ &+2\,b_k(\lambda)\,\psi_{2k}(\mu)\Bigg]\end{aligned} \tag{35}$$

This formula gives the mutual inductance of the two parallel circles of radii $a_1$ and $a_2$ where $a_2<a_1$ for the case where $a_1-a_2<r<a_1+a_2$. Here $r$ is the distance between centers, $\mu=\cos\theta$ where $\theta$ is the angle between the common direction of their axes and the line connecting their centers, and $\mu$ is to be taken as positive. The quantity $\lambda$ is $\left(\frac{a_1-a_2}{a_1+a_2}\right)^2$, and the function $b_k(\lambda)$ is to be computed by formula (13) and $\frac{\partial b_k(\lambda)}{\partial k}$ by formula (30); the $\psi$ function by (29) and the $\psi_{2k}(\mu)$ by either of the formulas (34) where $k$ is to be replaced by $2k$.

It is not difficult to show that the three expansions here obtained for $M$ make $M$ and its derivatives continuous at the spherical surfaces $r=a_1-a_2$ and $r=a_1+a_2$ and make $\frac{\partial M}{\partial \mu}$ reduce to $-2\pi r\sigma(r)$ when $\mu=+0$, and also make $M$ reduce in the first two cases, when $\mu=1$, to Maxwell's formula for coaxial circles. However, my direct proof that Maxwell's formula reduces to formula (35) when $\mu=1$ and $a_1-a_2<r<a_1+a_2$ is very complicated and is therefore left as an exercise for the reader. For the assistance of those interested, there follow a few relations valid for general value of $\nu$, which are derivable from the definition (13). They are

$$\left.\begin{aligned}&\sum_{k=-\infty}^{\infty}\frac{b_k(\lambda)}{k-\nu}=-\pi\cot\nu\pi\, b_\nu(\lambda) \qquad \nu \text{ not integral}\\ &\text{and}\\ &\sum_{k=0}^{\infty}\frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)}\frac{\lambda^k b_k(\lambda)}{k+\nu}=-\pi\frac{\Gamma(\nu)}{\Gamma\left(\nu-\frac{1}{2}\right)}b_\nu(\lambda)\end{aligned}\right\} \tag{36}$$

Also

$$g_\nu(\lambda)=\frac{4\pi(a_1+a_2)^{2\nu-1}}{\nu+\frac{1}{2}}\int_{a_1-a_2}^{a_1+a_2}\rho^{-2\nu}\sigma(\rho)\,d\rho \text{ if } \nu\neq-\frac{1}{2} \tag{37}$$

There is also a general relation for $0\leqq\mu\leqq1$ which may be useful

$$\sum_{n=0}^{\infty}(-1)^n\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(n+1)}P_{2n}(\mu)\frac{2n+\frac{1}{2}}{\left(n+\frac{\nu}{2}+\frac{1}{2}\right)\left(n-\frac{\nu}{2}\right)}= \tag{38}$$

$$=-\frac{\pi}{\sin\frac{\nu\pi}{2}}\frac{\Gamma\left(\frac{\nu}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{\nu}{2}+1\right)}P_\nu(\mu)$$

Returning to formula (35), consider the case where the circles are equal, $a_1 = a_2 = a$ and $\lambda = 0$. Then

$$b_k(0) = -2\sqrt{\pi}\frac{\Gamma\left(k-\frac{1}{2}\right)}{\Gamma(k+1)}$$

and

$$\frac{\partial b_k(0)}{\partial k} = b_k(0)\left[\psi\left(k-\frac{1}{2}\right)-\psi(k+1)\right] =$$

$$= -2\sqrt{\pi}\frac{\Gamma\left(k-\frac{1}{2}\right)}{\Gamma(k+1)}\left[2\log\frac{1}{2}-\frac{2}{2k-1}+\sum_{t=0}^{k-1\geq 0}\left(\frac{1}{t+\frac{1}{2}}-\frac{1}{t+1}\right)\right] \tag{39}$$

This gives

$$M(r,\mu) = 4a\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma\left(k-\frac{1}{2}\right)\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)\,\Gamma(k+1)}\left(\frac{r}{2a}\right)^{2k}\left\{\psi_{2k}(\mu)+\right.$$

$$\left.+P_{2k}(\mu)\left[\log\frac{r(\mu+1)}{16a}-\frac{2}{4k^2-1}+\sum_{t=0}^{k-1\geq 0}\left(\frac{1}{t+\frac{1}{2}}-\frac{1}{t+1}\right)\right]\right\} \tag{40}$$

The first four terms of this series were given by Butterworth in the paper mentioned. (Footnote 1.)

The formula for unequal circles which are coplanar and which intersect is obtained by placing $\mu = 0$ in equation (35). Since

$$P_{2\nu}(0) = \frac{\cos\nu\pi}{\sqrt{\pi}}\frac{\Gamma\left(\nu+\frac{1}{2}\right)}{\Gamma(\nu+1)}$$

it is found that

$$2\psi_{2k}(0) = P_{2k}(0)\left[2\log 2+\psi\left(k+\frac{1}{2}\right)-\psi(k+1)\right] \tag{41}$$

$$= P_{2k}(0)\sum_{t=0}^{k-1\geq 0}\left(\frac{1}{t+\frac{1}{2}}-\frac{1}{t+1}\right)$$

so that in this case

$$M(r,\,0) = -\frac{a_1+a_2}{2\pi}\sum_{k=0}^{\infty}\left(\frac{r}{a_1+a_2}\right)^{2k}\frac{\Gamma\left(k+\frac{1}{2}\right)\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)\,\Gamma(k+1)}\left\{\frac{\partial b_k(\lambda)}{\partial k}+\right.$$

$$\left.+2b_k(\lambda)\left[\log\frac{r}{4(a_1+a_2)}+\frac{1}{2k+1}+\sum_{t=0}^{k-1\geq 0}\left(\frac{1}{t+\frac{1}{2}}-\frac{1}{t+1}\right)\right]\right\} \tag{42}$$

Placing $a_1=a_2=a$ in this gives for equal coplanar, intersecting circles

$$M(r,0)=\frac{4a}{\sqrt{\pi}}\sum_{k=0}^{\infty}\left(\frac{r}{2a}\right)^{2k}\frac{\Gamma\left(k-\frac{1}{2}\right)\Gamma\left(k+\frac{1}{2}\right)\Gamma\left(k+\frac{3}{2}\right)}{\Gamma(k+1)\,\Gamma(k+1)\,\Gamma(k+1)}\left[\log\frac{r}{16a}\right.$$

$$\left.-\frac{2}{4k^2-1}+\frac{3}{2}\sum_{t=0}^{k-1\geq 0}\left(\frac{1}{t+\frac{1}{2}}-\frac{1}{t+1}\right)\right]. \tag{43}$$

which is also the value given by equation (40) for $\mu=0$.

## VII. SUMMARY

The formulas (23,) (25), and (35), therefore, give the mutual inductance of any two parallel circles. It is believed that the formula (35) is the first formula yet given for this complicated case of unequal circles. The fact that $\frac{\partial M}{\partial x}$ is not continuous when $x\to 0$ and $a_1-a_2<r<a_1+a_2$ shows the danger which lies in wait for one who constructs a solution of Laplace's equation and merely requires that it reduce to a known value upon a limited part of the axis of symmetry, without considering the boundary values at $x=0$.

WASHINGTON, February 20, 1929.

RP95

# SOIL-CORROSION STUDIES, 1927–28

By K. H. Logan


## ABSTRACT

This paper is the second progress report on the soil-corrosion studies, supplementing Technologic Paper No. 368[1] which set forth the plan of the investigation, described the soils and materials under test, and gave the data obtained up to and including 1926. The present report supplements the earlier one, gives additional data on the chemical properties of the soils and materials, and records the results of the examination of specimens removed in 1928. These specimens include only ferrous pipe materials and cable sheaths, no specimens of nonferrous materials other than cable sheaths, pipe coatings, or galvanized sheet being removed. The results of some laboratory and special field investigations are described. Soil characteristics, rather than differences in the composition of the ferrous pipe materials, appear to determine the type and extent of the corrosion observed. Tables show the relative corrosiveness of soils based on rate of loss of weight and rate of increase in depth of pits. Although it happened that the same soil was worst from both of these points of view, the soil that ranked next to the worst from the standpoint of pitting ranked twenty-eighth as to loss of weight.

As the specimens in the ground grew older the corrosion became more general, and in most soils there was a decrease in the rate of penetration. Field examination of cast-iron gas mains in several cities yielded data of a similar nature.

Preliminary studies indicated that, in addition to the highly localized galvanic action caused by mill scale, oxygen concentration cells, differences in soil contacts, etc., there are on long pipe lines galvanic currents that are the result of the line passing through different soils. There appeared to be a relation between the discharge of these galvanic currents and the corrosion observed on the pipe line.

Considerable work has been done recently on methods of predetermining the corrosiveness of soils. The problem has not been solved.

Additional specimens of pipe materials and coatings will probably be removed in 1930.


[1] B. S. Tech. Papers, 22, pp. 447–554, 1929.

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## I. INTRODUCTION

In April, 1928, the Bureau of Standards issued its first Technologic Paper[2] on its soil-corrosion investigation. That paper outlined the purpose and scope of the work then under way and gave such data concerning the soils, materials, and results of the work as were then available. The present paper is intended to supplement the earlier report and bring the results up to date. The work now under way will not be completed before 1935, and until then it is expected that progress reports will be issued from time to time as results become available. With respect to time, the investigation is now half completed, although but a little over a third of the 15,000 buried specimens have been removed.

The data discussed in the first technologic paper on soil-corrosion were obtained prior to 1927. Since that time considerable progress has been made. The Cast-Iron Pipe Research Association has employed an associate to assist in the study of soil corrosion, and the bureau has increased the number of men on this work. Assistance has also been secured from outside organizations, as will be noted in more detail later. As a result of this assistance, considerable progress has been made along several lines, but much remains to be done.

[2] B. S. Tech. Paper No. 368, Bureau of Standards Soil-Corrosion Studies. I. Soils, Materials, and Results of Early Observations.

The first conference on bituminous protective coatings for pipes[3] not only brought out the inadequacy of most of the coatings under test, but showed also the difficulty of determining the quality of a coating under consideration. It became apparent that a satisfactory study of protective coatings would involve extensive work in the laboratory to develop methods of testing coatings, as well as a large amount of field work to determine the character of coatings suitable for the varied conditions to which pipe lines are exposed. A little work has been done on the development of methods for testing protective coatings, and it is expected that this work will be continued during the coming year. Arrangements have been made for studies of protective coatings by research associates employed by the American Petroleum Institute and the American Gas Association.

TABLE 1.—*Results of soil analyses*

[In per cent of weight of dry soil]

| No. | Description | Loss by ignition | S | Insoluble | $Fe_2O_3$-$Al_2O_3$ | CaO | MgO | pH |
|---|---|---|---|---|---|---|---|---|
| | | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* |
| 1 | Allis silt loam | 7.06 | 0.08 | 83.8 | 8.4 | 0.32 | 0.08 | 7.0 |
| 2 | Bell clay | 10.40 | .05 | 82.6 | 4.6 | .54 | .32 | 7.4 |
| 3 | Cecil clay loam | 12.00 | .04 | 60.46 | 23.6 | .30 | .40 | 7.2 |
| 4 | Chester loam | 4.44 | .03 | 79.5 | 14.4 | .30 | .69 | ---- |
| 5 | Dublin clay adobe | 7.14 | .01 | 85.5 | 5.7 | .24 | .50 | 7.8 |
| 6 | Everett gravelly sandy loam | 3.40 | .03 | 87.6 | 7.1 | .40 | .58 | 6.9 |
| 7 | Fairmount silt loam | 11.72 | .04 | 83.5 | 4.1 | .30 | .29 | 7.0 |
| 8 | Fargo clay loam | 16.70 | .11 | 64.2 | 8.4 | 6.80 | 2.68 | 8.4 |
| 9 | Genesee silt loam | 7.26 | .03 | 88.4 | 3.1 | .30 | .40 | 8.6 |
| 12 | Hanford fine sandy loam | 3.48 | .04 | 87.8 | 5.9 | .40 | .94 | 7.4 |
| 13 | Hanford very fine sandy loam | 4.74 | .03 | 83.5 | 8.0 | 1.10 | 1.16 | ---- |
| 14 | Hempstead silt loam | 2.74 | .05 | 89.8 | 4.46 | 1.53 | .83 | 7.6 |
| 15 | Houston black clay | 15.00 | .08 | 74.0 | 5.40 | 3.58 | 1.00 | 8.4 |
| 16 | Kalmia fine sandy loam | 3.60 | .11 | 90.4 | 3.20 | .34 | .33 | 7.0 |
| 18 | Knox silt loam | 4.60 | .07 | 86.4 | 6.5 | .30 | .50 | 8.6 |
| 19 | Lindley silt loam | 5.80 | .07 | 84.3 | 7.9 | .30 | .80 | 6.9 |
| 20 | Mahoning silt loam | 6.00 | .06 | 81.6 | 10.80 | .30 | .72 | ---- |
| 21 | Marshall silt loam | 6.60 | .15 | 89.2 | 3.20 | .30 | .43 | 7.2 |
| 22 | Memphis silt loam | 5.00 | .06 | 88.0 | 4.80 | .30 | .47 | 6.8 |
| 25 | Miami clay loam | 20.40 | .08 | 49.3 | 14.80 | 11.80 | 2.82 | 8.0 |
| 26 | Miami silt loam | 19.34 | .09 | 51.24 | 17.30 | 10.96 | 1.52 | 8.2 |
| 27 | Miller clay | 12.98 | .08 | 81.40 | 2.00 | 1.80 | 1.30 | 8.4 |
| 28 | Montezuma clay adobe | 3.60 | .08 | 93.0 | 2.80 | .20 | .29 | 7.4 |
| 29 | Muck | 28.40 | .24 | 62.0 | 7.50 | .80 | .69 | 7.0 |
| 30 | Muscatine silt loam | 6.80 | .03 | 82.4 | 7.90 | .80 | .76 | 7.6 |
| 31 | Norfolk sand | .76 | .03 | 98.6 | 1.00 | .20 | .14 | 6.9 |
| 32 | Ontario loam | 4.60 | .06 | 85.3 | 6.40 | 2.00 | 1.21 | 8.4 |
| 33 | Peat | 63.22 | .04 | 22.1 | 4.40 | 6.80 | 2.30 | 8.6 |
| 34 | Penn silt loam | 4.40 | .03 | 84.8 | 9.10 | .40 | .62 | 7.4 |
| 35 | Ramona loam | 6.30 | .03 | 90.1 | 2.46 | .78 | .80 | 8.0 |
| 36 | Ruston sandy loam | 4.86 | .04 | 81.9 | 10.20 | .40 | .40 | 7.0 |
| 37 | St. Johns fine sand | 4.54 | .03 | 93.0 | 2.00 | .30 | .10 | 6.9 |
| 38 | Sassafras gravelly sandy loam | .20 | .05 | 96.7 | 3.20 | .10 | .12 | 7.0 |
| 39 | Sassafras silt loam | 7.00 | .04 | 92.8 | 1.70 | .30 | .40 | 7.0 |
| 40 | Sharkey clay | 7.90 | .06 | 83.6 | 6.70 | .40 | .59 | 8.0 |
| 41 | Summit silt loam | 8.56 | .21 | 84.62 | 4.68 | .30 | .65 | 7.4 |
| 42 | Susquehanna clay | 7.60 | .34 | 79.90 | 10.70 | .34 | .56 | ---- |
| 43 | Tidal marsh | 15.80 | 2.21 | 71.22 | 9.46 | .56 | 1.07 | ---- |
| 44 | Wabash silt loam | 8.20 | .42 | 87.58 | 2.40 | .34 | .70 | ---- |
| 45 | Alkali soil | 6.80 | 1.00 | 86.00 | 1.20 | 2.54 | .33 | 8.4 |
| 46 | Sandy loam | .64 | .18 | 95.22 | 1.40 | .30 | .00 | 8.6 |
| 47 | Silt loam | 21.52 | .24 | 44.70 | 9.16 | 22.00 | 2.27 | 8.6 |

NOTE.—Leaders in figure columns indicate that determination was impossible. Slight inaccuracies in some of the water analyses are due to the extremely small amount of solution available for some determinations.

[3] See B. S. Tech. Paper No. 368, p. 521.

TABLE 2.—*Results of analyses of solution*

[By parts per million of soil]

| Soil No. | $CO_3$ | $HCO_3$ | Cl | $SO_4$ | $SiO_2$ | $Fe_2O_3$+ $Al_2O_3$ | Ca | Mg | Na+K | Total solids |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 4 | 20 | 75 | 35 | 7 | 15 | 20 | 12 | 188 |
| 2 | 0 | 4 | 12 | 90 | 22 | 8 | 70 | 3 | 7 | 216 |
| 3 | 0 | 2 | 10 | 10 | 4 | 6 | 8 | 2 | 8 | 50 |
| 4 | 0 | 2 | 8 | ------ | 13 | 3 | 7 | 6 | ------ | 110 |
| 5 | 0 | 30 | 60 | 50 | 145 | 44 | 10 | 6 | 35 | 370 |
| 6 | 0 | 2 | 8 | 20 | 17 | 10 | 7 | 4 | 37 | 105 |
| 7 | 0 | 2 | 12 | 50 | 32 | 4 | 22 | 9 | 17 | 248 |
| 8 | 0 | 14 | 60 | 2, 120 | 27 | 0 | 6 | 484 | 169 | 2, 907 |
| 9 | 0 | 24 | 8 | 80 | 24 | 5 | 105 | 15 | 7 | 268 |
| 12 | 0 | 4 | 6 | ------ | 19 | 3 | 26 | 18 | ------ | 230 |
| 13 | ------ | ------ | ------ | 95 | 68 | 301 | 130 | 3 | ------ | 4, 755 |
| 14 | 0 | 4 | 6 | 40 | 16 | 5 | 36 | 8 | 10 | 126 |
| 15 | 0 | 10 | 22 | 385 | 20 | 5 | 46 | 7 | 140 | 630 |
| 16 | 0 | 4 | 4 | 40 | 100 | 40 | 23 | 36 | 8 | 256 |
| 18 | 0 | 16 | 12 | 40 | 21 | 10 | 41 | 15 | 2 | 157 |
| 19 | 0 | 2 | 4 | 30 | 20 | 12 | 18 | 11 | 8 | 105 |
| 20 | 0 | 4 | 12 | 100 | 90 | 26 | 35 | 13 | 0 | 255 |
| 21 | 0 | 4 | 12 | 30 | 26 | 6 | 26 | 7 | 4 | 115 |
| 22 | 0 | 2 | 6 | 50 | 104 | 40 | 15 | 9 | 6 | 232 |
| 25 | 0 | 12 | 8 | 10 | 22 | 5 | 48 | 15 | 2 | 122 |
| 26 | 0 | 10 | 50 | 20 | 15 | 5 | 62 | 16 | 2 | 180 |
| 27 | 0 | 12 | 18 | ------ | 35 | 5 | 12 | 39 | 344 | 455 |
| 28 | 0 | 11 | 218 | 300 | 42 | 13 | 49 | 25 | 298 | 955 |
| 29 | 0 | 4 | 16 | 195 | 35 | 8 | 118 | 13 | 6 | 395 |
| 30 | 0 | 6 | 14 | 20 | 2 | 15 | 26 | 20 | 4 | 107 |
| 31 | 0 | 4 | 6 | 20 | 18 | 17 | 14 | 9 | 46 | 134 |
| 32 | 0 | 14 | 4 | 60 | 15 | 8 | 49 | 16 | 20 | 186 |
| 33 | 0 | ------ | ------ | ------ | 15 | 20 | 28 | 118 | ------ | 4, 062 |
| 34 | 0 | 6 | 6 | 35 | 15 | 11 | 17 | 10 | 6 | 106 |
| 35 | 0 | 10 | 18 | 80 | 480 | 170 | 14 | 24 | 9 | 815 |
| 36 | 0 | 2 | 136 | 20 | 25 | 10 | 16 | 18 | 38 | 265 |
| 37 | 0 | 4 | 10 | 20 | 20 | 10 | 10 | 8 | 3 | 85 |
| 38 | 0 | 4 | 2 | 20 | 10 | 15 | 5 | 6 | 0 | 60 |
| 39 | 0 | 4 | 4 | 50 | 10 | 12 | 7 | 2 | 9 | 98 |
| 40 | 0 | 10 | 6 | 20 | 30 | 5 | 6 | 9 | 6 | 92 |
| 41 | 0 | 2 | 4 | 20 | 25 | 45 | 14 | 10 | 2 | 122 |
| 42 | 0 | 4 | 6 | 4 | 8 | 10 | 7 | 4 | 0 | 35 |
| 43 | 0 | ------ | ------ | 5, 850 | 40 | 1, 516 | 217 | 231 | ------ | 13, 780 |
| 44 | 0 | 4 | 8 | 160 | 13 | 10 | 37 | 17 | 233 | 472 |
| 45 | 0 | ------ | ------ | 3, 085 | ------ | ------ | ------ | ------ | ------ | ------ |
| 46 | 0 | 14 | 14 | 100 | 10 | 5 | 50 | 8 | 9 | 210 |
| 47 | 0 | 20 | 6 | 20 | 13 | 15 | 35 | 14 | 7 | 130 |

NOTE.—Leaders in figure columns indicate that determination was impossible. Slight inaccuracies in some of the water analyses are due to the extremely small amount of solution available for some determinations.

## II. ADDITIONAL DATA ON SOILS

There is a belief on the part of many that soil corrosion is closely related to the chemical nature of the soil. We were fortunate, therefore, in securing the cooperation of J. W. Richards & Son, industrial chemists, who have made the analyses shown in Tables 1 and 2 of the soils in which specimens have been buried. Their method of preparing the water extract in which the salt content of the soil was determined, as described by Percy J. Richards, was as follows: "Six ounces of the soil sample are mixed with 300 cc of boiled distilled water in a small bottle; the contents should be thoroughly

agitated at least 10 times a day for six days * * * then filter into a clean bottle and cork for analysis."

When the rates of corrosion of the specimens have been more definitely established by the examination of more specimens, these tables may be very helpful in the study of the relation of the chemical nature of the soil to corrosion. At the present time, such a relation is not well established. It is not to be expected that a knowledge of the constituents of the soil will in itself make possible a prediction as to the rate of corrosion, since the supply of oxygen and water may be factors. The amounts of oxygen and water reaching the specimens depend upon the position of the trench in which specimens are buried, the type of soil, and the amount and distribution of rainfall. The soil descriptions given in Technologic Paper No. 368 include the soil characteristics and drainage, while the rainfall and temperature can be judged in a general way by the location of the specimens, or more detailed data can be secured from the reports of the Weather Bureau. It has been suggested that weather conditions for the months immediately preceding and following the burial of the specimens may have affected the type of initial corrosion product formed and, consequently, the subsequent rate of corrosion. Plans for a study of this question are under way.

The amount of colloidal matter in a soil may have a bearing on corrosion because of the relation of colloids to the ability of the soil to absorb and hold water and because of the change in volume of soils containing large amounts of colloids with change in moisture content. Information on the colloid content, as determined by the method of Bouyoucos,[4] is given in Table 3.

TABLE 3.—*Soil colloids* [a]

| Soil No. | Per cent | Soil No. | Per cent | Soil No. | Per cent | Soil No. | Per cent |
|---|---|---|---|---|---|---|---|
| 1 | 63.6 | 13 | 17.8 | 25 | 31.0 | 37 | 6.6 |
| 2 | 46.6 | 14 | 32.0 | 26 | 50.1 | 38 | 5.5 |
| 3 | 43.3 | 15 | 63.1 | 27 | 91.4 | 39 | 15.5 |
| 4 | 25.3 | 16 | 20.8 | 28 | 29.1 | 40 | 49.8 |
| 5 | 44.8 | 17 | 76.0 | 29 | 24.5 | 41 | 44.0 |
| 6 | 11.1 | 18 | 28.8 | 30 | 41.6 | 42 | 44.6 |
| 7 | 70.3 | 19 | 38.9 | 31 | 1.1 | 43 | 51.0 |
| 8 | 66.0 | 20 | 43.6 | 32 | 22.7 | 44 | 31.5 |
| 9 | 35.9 | 21 | 42.8 | 33 | 20.4 | 45 | |
| 10 | 15.9 | 22 | 33.2 | 34 | 41.9 | 46 | 7.0 |
| 11 | 52.5 | 23 | 17.9 | 35 | 20.0 | 47 | 53.6 |
| 12 | 8.9 | 24 | 7.2 | 36 | 37.9 | | |

[a] Determinations by Dr. S. P. Ewing, Bureau of Standards.

[4] The Hydrometer as a New and Rapid Method for Determining the Colloidal Content of Soils, B. J. Bouyoucos, Soil Science, **23**, No. 4, p. 319.

## III. RESULTS OF ANALYSES OF FERROUS PIPE MATERIALS

Earlier reports have stated that the several kinds of ferrous pipe materials in any one soil showed similar corrosion patterns and that for any one material this pattern varied with the soil. The indications were, therefore, that the initial corrosion was determined largely by soil characteristics.

There were, however, minor differences, which appeared to be caused by differences in the pipe materials, and these may become more pronounced on the specimens which are exposed longer. For this reason the character of the material under test is of interest. Since the specimens were selected from stock, not all of the specimens of any one material were necessarily produced at the same time, and, consequently, they may differ somewhat in composition. An attempt has been made to gain some idea of the character of the materials by analysis of a few specimens. Most of these analyses were furnished by the manufacturer at the time he furnished specimens. In a few places, as indicated in Table 4, the analyses were made by the A. O. Smith Corporation, who analyzed the other rolled specimens also.

TABLE 4.—*Results of chemical analyses of pipe materials in the Bureau of Standards soil-corrosion investigation*

| Diameter of pipe, inches | Material | Number of specimens tested | C | Mn | P | S | Si | Cu | Analysis by— |
|---|---|---|---|---|---|---|---|---|---|
| | | | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* | |
| 1½ | Open-hearth iron | 2 | 0.02 | Trace. | 0.010 | 0.050 | 0.09 | 0.014 | [1] |
| 1½ | Wrought iron | 4 | .03 | Trace. | .145 | .023 | .15 | .02 | [1] |
| 1½ | Bessemer steel | 16 | .09 | 0.39 | .088 | .040 | | | [2] |
| 1½ | Bessemer steel treated | 49 | | .38 | .092 | .050 | | | [2] |
| 3 | Bessemer steel | 1 | .08 | .40 | .098 | .038 | | | [2] |
| 3 | Open-hearth steel | 6 | .12 | .41 | .043 | .036 | | | [2] |
| 3 | Open-hearth steel+Cu | ? | .07 | .24 | .008 | .032 | | .22 | [2] |
| 3 | High-silicon cast iron | 1 | .72 | .26 | .11 | .123 | 13.44 | | [3] |
| 6 | De Lavaud cast iron | 2 | 3.56 | .73 | .77 | .083 | 2.34 | | [2] |
| 6 | Cast iron (southern) | 3 | 3.40 | .43 | .84 | .083 | 1.61 | | [2] |
| 6 | Cast iron (northern) | 2 | 3.45 | .56 | .55 | .075 | 1.55 | | [2] |

[1] A. O. Smith Corporation.
[2] Manufacturer furnishing material.
[3] United States Cast-Iron Pipe & Foundry Co.

## IV. TREND OF THE RESULTS OF FIELD TESTS OF PIPE SPECIMENS

The soil-corrosion specimens removed in 1928 were limited to specimens of pipe and cable sheath buried in 1922. The specimens have been cleaned and the depth of pits measured. Data on each individual specimen can be supplied to those needing these details, but for most purposes the average results presented in Tables 5, 6, and 7 are sufficient.

Space limitations make it impracticable to include the names of the soils in these tables. The names and locations of the soils cor-

responding to the soil numbers in Tables 5, 6, and 7 will be found in Table 8. It should be understood that the soil under test in or near a given city is not necessarily characteristic of the soil of that city; for example, there is little Cecil clay loam to be found in the built-up section of the city of Atlanta, Ga., and there is very little Susquehanna clay within the city limits of Meridian, Miss.

Since the preparation of the progress reports presented at the meetings of the American Foundrymen's Association in 1925 and 1927, several soils have been reidentified, and this has resulted in a rearrangement of the alphabetical list of soils. As these progress reports had rather wide circulation, it appears desirable to indicate the change in soil numbers, and this is done in Table 9.

Comparison of the rates of corrosion and pitting for the 2, 4, and 6 year periods are of interest. In comparing results in different soils it must be kept in mind that the periods of burial, especially the first periods, are somewhat different in different parts of the country. In most cases, the rates of corrosion and pitting are greatest for the first period.

### 1. RATES OF CORROSION

Since specimens of De Lavaud cast iron were not buried in all of the test locations until some time after the other specimens were buried, they were not removed in 1928 from some of the soils. Moreover, the data on loss of weight of the first specimens buried were based on corrosion on both the inside and the outside surfaces of the pipe, as was the case with all other specimens. While the two surfaces of the other specimens were substantially the same, the outer surface of De Lavaud cast iron differs materially from the inside surface, as was evidenced from the appearance of the surfaces after corrosion. For these reasons we have not included the data on De Lavaud cast iron in summarizing the data.

If we take the data on four or more of the six classes of material in Table 5 in any one soil (De Lavaud cast iron being disregarded) as indicative of the trend of the corrosion in that soil, we find that in 17 out of the 45 soils from which specimens were removed in 1928 there has been a decrease in the rate of corrosion throughout the period of the investigation. In 5 other soils the rate for the 1928 specimens was less than that for the specimens removed in 1926, and in 10 other soils the rate for the third period was less than that for the first period. In only 2 soils was there a progressive increase in the rate of corrosion. In 11 soils the rate for 1928 was greater than for 1926. In a considerable number of cases the variations in the apparent rates seemed due to accidental occurrences incident to conditions at time of burial. If we average the results in all soils, we find that for each material there was a continuous decrease in the rate of corrosion.

TABLE 5.—*Comparison of rates of corrosion*[1] *for three periods*

| Soil No. | Time buried (years) | Bessemer steel (6 specimens) | | | Wrought iron (4 specimens) | | | Pure open-hearth iron (2 specimens) | | | Open-hearth steel (2 specimens) | | | Copper-bearing steel (2 specimens) | | | Sand mold cast iron (2 specimens) | | | De Lavaud cast iron (2 specimens) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third |
| 1 | 0.98 | 3.58 | 5.50 | 1.19 | 1.12 | 0.93 | 1.23 | 1.16 | 0.99 | 1.08 | 0.92 | 0.81 | 1.39 | 1.17 | 0.97 | 1.17 | 0.96 | 0.84 | 1.45 | 1.05 | 1.05 | 0.80 | 0.96 | 0.85 |
| 2 | 2.09 | 4.03 | 5.93 | 1.03 | .75 | .65 | .93 | .80 | .70 | .90 | .71 | .64 | 1.05 | .68 | .68 | .88 | .62 | .71 | .93 | .97 | .89 | .43 | (3) | (3) |
| | [2] 1.94 | | | | | | | | | | | | | | | | | | | | | | | |
| | 2.01 | 4.09 | 6.04 | .97 | .84 | .60 | .85 | .83 | .66 | .73 | .70 | .52 | .98 | .81 | .58 | .86 | .89 | .69 | .98 | 1.03 | .69 | (3) | (3) | |
| 3 | [2] 2.07 | | | | | | | | | | | | | | | | | | | | | .58 | | |
| 4 | 1.38 | 4.03 | 6.12 | 1.04 | .87 | .73 | .92 | .90 | .74 | 1.06 | .91 | .73 | 1.01 | .91 | .75 | 1.12 | .92 | .80 | .99 | 1.01 | .73 | .91 | .90 | .73 |
| 5 | 1.93 | 4.13 | 6.16 | .61 | .57 | .76 | .49 | .53 | .81 | .57 | .35 | .79 | .51 | .51 | .64 | .49 | .59 | .72 | .46 | .42 | .78 | (3) | (3) | |
| | [2] 2.20 | | | | | | | | | | | | | | | | | | | | | 1.17 | | |
| 6 | 1.93 | 4.13 | 6.17 | .09 | .18 | .13 | .07 | .15 | .10 | .07 | .17 | .10 | .07 | .17 | .13 | .08 | .13 | .12 | .06 | .08 | .08 | (3) | (3) | |
| | [2] 2.20 | | | | | | | | | | | | | | | | | | | | | .12 | | |
| 7 | 1.04 | 3.50 | (2) | .61 | .70 | (3) | .62 | .73 | (3) | .46 | .70 | (3) | .51 | .83 | (3) | .59 | .72 | (3) | 1.25 | .84 | (3) | .60 | .76 | (3) |
| 8 | 1.13 | 3.80 | 5.83 | .67 | .51 | .51 | .59 | .47 | .52 | .64 | .50 | .54 | .54 | .45 | .53 | .64 | .48 | .57 | 1.24 | 1.06 | .92 | .69 | .49 | .58 |
| 9 | 1.03 | 3.48 | 5.47 | .73 | .59 | .45 | .86 | .65 | .45 | .54 | .49 | .40 | .61 | .35 | .49 | .52 | .49 | .47 | .69 | .66 | .57 | .70 | .41 | .52 |
| 10 | 1.33 | 4.01 | 6.10 | .98 | .49 | .56 | .89 | .45 | .56 | 1.01 | .34 | .56 | .78 | .30 | .60 | .88 | .39 | .56 | .71 | .54 | .45 | .75 | .43 | .50 |
| 11 | 1.36 | 3.97 | 6.01 | .33 | .27 | .22 | .30 | .26 | .23 | .20 | .23 | .17 | .29 | .23 | .23 | .26 | .24 | .26 | .17 | .22 | .19 | .07 | .16 | .10 |
| 12 | 1.94 | 4.14 | 6.17 | .17 | .39 | .49 | .11 | .41 | .45 | .13 | .34 | .52 | .11 | .37 | .44 | .13 | .39 | .47 | .13 | .38 | .36 | (3) | (3) | |
| | [2] 2.21 | | | | | | | | | | | | | | | | | | | | | .03 | | |
| 13 | 1.93 | 4.22 | 5.89 | 2.12 | .92 | 1.24 | 1.76 | 1.18 | 1.17 | 1.28 | .64 | 1.11 | 2.33 | .55 | 1.04 | 2.11 | 1.11 | 1.45 | 1.90 | 1.99 | 2.16 | 1.41 | .68 | 1.12 |
| 14 | 1.13 | 3.80 | 5.83 | .42 | .63 | .56 | .32 | .62 | .61 | .23 | .52 | .42 | .41 | .59 | .55 | .40 | .51 | .50 | .26 | .60 | .80 | .24 | .30 | .37 |
| 15 | 1.96 | 4.04 | 5.93 | 1.21 | .83 | .91 | 1.10 | .75 | 1.04 | 1.25 | .89 | 1.04 | 1.15 | .83 | .95 | 1.38 | .76 | 1.08 | .97 | .83 | 1.22 | (3) | (3) | |
| | [2] 2.08 | | | | | | | | | | | | | | | | | | | | | 1.15 | | |
| 16 | 1.97 | 4.04 | 5.97 | 1.25 | .81 | .77 | 1.17 | .83 | .87 | .96 | .71 | .88 | 1.14 | .74 | .75 | 1.06 | .85 | .83 | 1.25 | .59 | .98 | (3) | (3) | |
| | [2] 2.07 | | | | | | | | | | | | | | | | | | | | | .76 | | |
| 17 | 1.21 | 3.83 | 5.87 | 1.23 | .90 | .96 | 1.27 | .90 | 1.02 | 1.22 | .86 | .88 | 1.24 | .90 | 1.04 | 1.21 | .95 | 1.06 | 1.94 | .96 | 1.05 | 1.15 | .94 | .83 |
| 18 | 1.19 | 3.74 | 5.78 | .42 | .42 | .52 | .47 | .42 | .60 | .35 | .32 | .39 | .27 | .30 | .47 | .40 | .44 | .44 | (3) | .27 | .72 | .21 | .16 | .39 |
| 19 | 1.12 | 3.66 | 5.70 | .63 | .57 | .39 | .65 | .51 | .46 | .56 | .54 | .38 | .56 | .62 | .40 | .75 | .55 | .37 | .49 | .35 | .33 | .34 | .37 | .37 |
| 20 | .98 | 3.57 | 5.50 | 1.05 | .58 | .44 | .90 | .64 | .45 | .95 | .60 | .44 | 1.05 | .60 | .41 | .95 | .56 | .41 | 1.11 | .67 | .68 | .75 | .51 | .57 |
| 21 | 1.47 | 4.02 | 6.05 | 1.43 | .68 | .78 | 1.45 | .67 | .76 | 1.37 | .63 | .70 | 1.35 | .73 | .84 | 1.44 | .62 | .76 | 1.53 | .79 | .93 | 1.09 | .62 | .73 |
| 22 | 1.72 | 3.68 | 5.55 | 1.03 | .96 | .84 | .88 | 1.01 | .90 | .82 | .87 | .95 | .89 | 1.00 | .83 | 1.03 | .94 | .64 | .91 | 1.11 | .81 | .62 | .67 | .67 |
| 23 | 1.93 | 4.27 | 6.16 | 3.85 | 2.82 | 2.26 | 3.48 | 2.95 | 2.32 | 2.80 | 2.62 | 2.22 | 3.38 | 2.79 | 2.35 | 4.06 | 2.26 | 2.83 | 7.11 | 6.74 | 5.75 | 2.96 | 2.89 | 2.88 |
| 24 | 1.32 | 4.01 | 6.10 | .21 | .11 | .18 | .17 | .12 | .19 | .15 | .11 | .14 | .20 | .09 | .18 | .15 | .08 | .16 | .11 | .09 | .13 | .13 | .05 | .14 |
| 25 | 1.03 | 3.71 | 5.74 | .42 | .28 | .32 | .45 | .28 | .30 | .43 | .19 | .29 | .36 | .23 | .29 | .40 | .28 | .26 | .48 | .45 | .34 | .47 | .30 | .22 |
| 26 | 1.04 | 3.49 | 5.48 | .81 | .56 | .27 | .84 | .60 | .30 | .76 | .45 | .26 | .73 | .55 | .21 | .87 | .54 | .24 | 1.04 | .95 | .41 | .76 | .56 | .33 |
| 27 | 1.95 | 4.05 | 5.96 | .26 | .89 | .57 | .31 | .82 | .58 | .20 | .80 | .50 | .29 | .86 | .51 | .24 | .66 | .53 | .24 | .79 | .65 | 1.10 | (3) | (3) |
| | [2] 2.13 | | | | | | | | | | | | | | | | | | | | | | | |
| 28 | 1.64 | (3) | (2) | 1.92 | (3) | (3) | 1.76 | (3) | (3) | 2.04 | (3) | (3) | 1.94 | (3) | (3) | 2.15 | (3) | (3) | 2.86 | (3) | (3) | 2.76 | (3) | (3) |
| 29 | 1.96 | 4.07 | 5.99 | 2.09 | 1.73 | 1.57 | 1.74 | 1.84 | 1.51 | 2.03 | 1.78 | 1.67 | 2.16 | 1.79 | 1.61 | 2.13 | 1.94 | 1.56 | 1.99 | 2.05 | 1.80 | 3.24 | 3.45 | 3.52 |
| | | | | | | | | | | | | | | | | | | | | | | 1.04 | | |

[1] Logan, Proc. Am. Soc. Test. Mat., 17, II, 1917.
[2] Duplicate.
[3] Lost.

| | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 28 | 1.64 | (3) | (3) | 1.92 | (3) | (3) | 1.76 | (3) | (3) | 2.04 | (3) | (3) | 1.94 | (3) | (3) | 2.15 | (3) | (3) | 2.86 | (3) | (3) | 2.76 | (3) | (3) |
| 29 | 1.96 | 4.07 | 5.99 | 2.09 | 1.73 | 1.57 | 1.74 | 1.84 | 1.51 | 2.03 | 1.78 | 1.67 | 2.16 | 1.79 | 1.61 | 2.13 | 1.94 | 1.56 | 1.99 | 2.05 | 1.80 | 3.24 | (3) | (3) |
| 30 | 1.11 | 3.65 | 5.69 | .94 | .35 | .42 | .81 | .36 | .42 | .81 | .30 | .32 | .96 | .35 | .47 | .91 | .34 | .41 | 1.12 | .51 | .49 | 1.04 | .45 | .52 |
| 31 | [1] 2.01 | 4.07 | 6.02 | .68 | .53 | .33 | .51 | .46 | .27 | .34 | .50 | .25 | .62 | .62 | .31 | .52 | .56 | .39 | .37 | .45 | .30 | 1.09 | (3) | (3) |
| | [2] 2.07 | | | | | | | | | | | | | | | | | | | | | | | |
| 32 | .94 | 3.73 | 5.82 | .40 | .39 | .38 | .42 | .42 | .37 | .27 | .31 | .30 | .42 | .35 | .38 | .34 | .38 | .43 | .40 | .53 | .48 | .52 | .42 | .40 |
| 33 | 1.03 | 3.71 | 5.75 | .41 | .94 | .87 | .39 | .95 | .77 | .42 | .94 | .81 | .44 | .93 | .97 | .35 | 1.02 | .52 | .35 | .88 | .38 | .42 | .92 | .58 |
| 34 | 1.38 | 4.03 | 6.12 | .89 | .45 | .45 | .96 | .45 | .45 | .95 | .40 | .50 | .98 | .46 | .51 | .86 | .44 | .48 | .95 | .45 | .47 | .91 | .43 | .46 |
| 35 | [1] 1.93 | 4.14 | 6.16 | .44 | .41 | .19 | .43 | .50 | .17 | .42 | .41 | .18 | .47 | .47 | .11 | .48 | .43 | .22 | .60 | .70 | .20 | .51 | (3) | (3) |
| | [2] 2.20 | | | | | | | | | | | | | | | | | | | | | | | |
| 36 | [1] 1.99 | 4.07 | 5.97 | .58 | .49 | .27 | .52 | .45 | .26 | .58 | .35 | .22 | .42 | .42 | .31 | .47 | .37 | .26 | .47 | .32 | .27 | .36 | (3) | (3) |
| | [2] 2.08 | | | | | | | | | | | | | | | | | | | | | | | |
| 37 | [1] 2.00 | 4.07 | 6.03 | 1.21 | 1.08 | .68 | 1.17 | 1.09 | .70 | 1.15 | 1.03 | .69 | 1.20 | 1.09 | .69 | 1.21 | 1.21 | .62 | .83 | 1.00 | .88 | 1.95 | (3) | (3) |
| | [2] 2.07 | | | | | | | | | | | | | | | | | | | | | | | |
| 38 | 1.40 | 4.04 | 6.13 | .19 | .15 | .18 | .13 | .15 | .15 | .11 | .11 | .14 | .18 | .10 | .21 | .20 | .13 | .20 | .06 | .09 | .15 | .03 | .07 | .17 |
| 39 | 1.38 | 4.02 | 6.11 | .87 | .74 | .46 | .85 | .80 | .53 | .75 | .78 | .39 | .99 | .76 | .50 | .90 | .73 | .53 | 1.02 | .74 | .76 | .86 | .71 | .58 |
| 40 | [1] 1.96 | 4.06 | 5.98 | 1.07 | 1.00 | .92 | 1.12 | 1.04 | .97 | 1.09 | .97 | .89 | 1.28 | 1.05 | 1.10 | 1.45 | 1.02 | .94 | 1.17 | 1.42 | 1.54 | 1.12 | (3) | (3) |
| | [2] 2.10 | | | | | | | | | | | | | | | | | | | | | | | |
| 41 | 1.47 | 4.02 | 6.05 | .44 | .70 | .71 | .40 | .77 | .66 | .51 | .68 | .76 | .43 | .73 | .59 | .27 | .62 | .64 | .82 | .65 | .54 | .26 | .46 | .35 |
| 42 | 1.98 | 4.07 | 5.97 | 1.57 | 1.36 | 1.18 | 1.64 | 1.49 | 1.14 | 1.51 | 1.09 | 1.18 | 1.20 | .83 | 1.15 | .97 | .84 | 1.10 | 2.45 | 2.28 | 2.09 | 1.14 | (3) | (3) |
| 43 | 1.29 | 4.06 | 6.16 | 1.80 | 1.34 | 1.34 | 1.44 | 1.48 | 1.22 | 1.05 | .90 | .93 | 1.70 | 1.28 | 1.00 | 1.87 | 1.58 | 1.35 | .91 | .93 | 1.10 | .78 | .77 | .92 |
| 44 | 1.09 | 3.63 | 5.68 | .38 | .47 | .40 | .47 | .45 | .38 | .30 | .38 | .39 | .32 | .38 | .38 | .38 | .35 | .34 | .59 | .32 | .29 | .15 | .27 | .17 |
| 45 | 1.18 | 3.78 | 5.79 | 1.22 | .79 | .55 | 1.05 | .78 | .56 | 1.00 | .96 | .50 | .81 | .85 | .53 | 1.16 | .86 | .52 | 1.36 | 1.11 | .44 | 1.14 | 2.06 | .40 |
| 46 | 1.46 | 4.05 | 6.07 | .76 | .67 | .47 | .80 | .69 | .46 | .53 | .68 | .47 | .76 | .63 | .49 | .79 | .79 | .52 | 1.47 | 1.17 | .49 | .35 | .55 | .36 |
| 47 | 1.48 | 4.08 | 6.09 | .30 | .41 | .27 | .27 | .39 | .27 | .35 | .31 | .19 | .39 | .32 | .26 | .25 | .50 | .24 | .38 | .59 | .46 | .40 | .46 | .44 |
| Average | | | | .91 | .72 | .64 | .85 | .74 | .65 | .79 | .65 | .60 | .87 | .69 | .63 | .89 | .69 | .64 | 1.06 | .90 | .82 | .83 | .64 | .57 |

[1] Loss in ounces per square foot per year.

[2] Time for De Lavaud cast iron.

[3] Specimens not removed.

## 2. RATES OF PITTING

(*a*) METHOD OF DETERMINING RATES.—Table 6 shows the averages of the maximum rates of pitting for each class of material. The depth of the deepest pit on each specimen was first divided by the period the specimen was buried, then the average was obtained by dividing the sum of these quotients by the number of specimens in the class.

As the smoothness of specimens of commercial pipe was not sufficient to permit the accurate measurement of shallow pits, the rule was adopted that no record should be made of pits less than 0.02 inch in depth. When no specimen in a class of material in a given soil was pitted sufficiently to justify measurement, no pitting rate was given for this class in that soil. If only part of the specimens in a class were pitted deeply enough for measurement, it was necessary to make some assumption regarding the pitting of the other specimens if an average was to be obtained. It was decided, therefore, to assume that the total depth of the deepest pit on each unmeasured specimen was 0.02. While the resultant rates for these specimens were obviously somewhat too high, it was believed that they did not cause a serious error in the results, and for the sake of uniformity in method and comparability of results it was suggested that others who have occasion to compute average pitting data make the same assumptions. The depth of the pits was measured from a portion of the original surface preserved for this purpose.

TABLE 6.—*Comparison of rates of penetration for three periods*

TABLE 6.—*Comparison of rates of penetration[1] for three periods*

| Soil No. | Time buried (years) | | | Bessemer steel (6 specimens) | | | Wrought iron (4 specimens) | | | Pure open-hearth iron (2 specimens) | | | Open-hearth steel (2 specimens) | | | Copper-bearing steel (2 specimens) | | | Sand mold cast iron (2 specimens) | | | De Lavaud cast iron (2 specimens) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third |
| 1 | 0.98 | 3.58 | 5.50 | ------ | 11 | 9 | ------ | 10 | 9 | ------ | 13 | 10 | ------ | 10 | 10 | ------ | 13 | 9 | 45 | 29 | 20 | [3] 23 | 17 | 12 |
| 2 | 2.09 | 4.03 | 5.93 | ------ | 11 | 8 | ------ | 11 | 6 | ------ | 13 | 7 | ------ | 11 | 7 | ------ | 11 | 10 | 19 | 10 | 12 | [3] 10 | (4) | (4) |
| | [2] 1.94 | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 2.01 | 4.09 | 6.04 | 25 | 18 | 11 | 25 | 16 | 11 | 31 | 18 | 11 | 26 | 13 | 13 | 30 | 22 | 10 | 32 | 16 | 21 | ------ | (4) | (4) |
| | [2] 2.07 | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 1.38 | 4.03 | 6.12 | ------ | 7 | 7 | ------ | 6 | 8 | ------ | 8 | 13 | ------ | 8 | 9 | ------ | 10 | 15 | 24 | 12 | 7 | ------ | 8 | 10 |
| 5 | 1.93 | 4.13 | 6.16 | ------ | 8 | 8 | ------ | [3] 6 | 6 | ------ | 7 | 7 | ------ | 7 | 10 | ------ | 8 | 6 | [3] 14 | 9 | 12 | ------ | (4) | (4) |
| | [2] 2.20 | | | | | | | | | | | | | | | | | | | | | 19 | | |
| 6 | 1.93 | 4.13 | 6.17 | ------ | [3] 5 | [3] 3 | ------ | [3] 5 | [3] 3 | ------ | [3] 5 | 4 | ------ | [3] 5 | [3] 3 | ------ | ------ | [3] 3 | ------ | ------ | ------ | ------ | (2) | (4) |
| | [2] 2.20 | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | 1.04 | 3.50 | (4) | ------ | [3] 7 | (4) | ------ | [3] 6 | (4) | ------ | 8 | (4) | ------ | 9 | (4) | ------ | [3] 7 | (4) | ------ | [3] 8 | (4) | ------ | [3] 8 | (4) |
| 8 | 1.13 | 3.80 | 5.83 | 34 | 11 | 9 | 30 | 10 | 10 | 38 | 12 | 10 | 38 | 13 | 13 | 54 | 15 | 12 | 59 | 18 | 15 | ------ | [3] 10 | 11 |
| 9 | 1.03 | 3.48 | 5.47 | 24 | 17 | 5 | 38 | 15 | 5 | 44 | 13 | [3] 4 | 31 | 17 | 8 | 32 | 18 | 6 | ------ | 10 | [3] 5 | ------ | [3] 10 | 8 |
| 10 | 1.33 | 4.01 | 6.10 | 14 | [3] 5 | 7 | 16 | [3] 6 | 7 | 19 | ------ | 7 | 15 | ------ | 6 | 19 | ------ | 8 | ------ | ------ | 9 | [3] 18 | ------ | 6 |
| 11 | 1.36 | 3.97 | 6.01 | 24 | 16 | 12 | 27 | 17 | 11 | 36 | 16 | 11 | 24 | 13 | 10 | 31 | 15 | 12 | [3] 22 | 9 | 7 | 26 | 6 | [3] 4 |
| 12 | 1.94 | 4.14 | 6.17 | ------ | 11 | 10 | ------ | 10 | 10 | ------ | 10 | 9 | ------ | 9 | 9 | ------ | 13 | 10 | ------ | 11 | 7 | ------ | (4) | (4) |
| | [2] 2.21 | | | | | | | | | | | | | | | | | | | | | | | |
| 13 | 1.93 | 4.22 | 5.89 | 25 | 11 | 13 | 20 | 11 | 13 | 18 | 6 | 8 | 33 | 13 | 11 | 37 | 15 | 9 | [3] 21 | 35 | 37 | 16 | 13 | [3] 4 |
| 14 | 1.13 | 3.80 | 5.83 | 28 | 14 | 15 | 31 | 16 | 15 | 29 | 14 | 15 | 28 | 15 | 15 | 57 | 19 | 26 | 24 | [3] 7 | 7 | ------ | [3] 11 | 4 |
| 15 | 1.96 | 4.04 | 5.93 | 20 | 9 | 9 | 19 | 7 | 10 | 15 | 11 | 11 | 17 | 8 | 11 | 22 | 12 | 10 | 40 | 12 | 20 | ------ | (4) | (4) |
| | [2] 2.08 | | | | | | | | | | | | | | | | | | | | | 14 | | |
| 16 | 1.97 | 4.04 | 5.97 | 20 | 14 | 17 | 19 | 12 | 14 | 21 | 11 | 15 | 17 | 11 | 10 | 22 | 17 | 15 | 23 | 14 | 18 | ------ | (4) | (4) |
| | [2] 2.07 | | | | | | | | | | | | | | | | | | | | | 15 | | |
| 17 | 1.21 | 3.83 | 5.87 | ------ | 6 | 5 | ------ | 6 | 5 | ------ | 5 | 5 | ------ | 6 | 5 | ------ | [3] 6 | 6 | 39 | 6 | 11 | ------ | [3] 6 | 8 |
| 18 | 1.19 | 3.74 | 5.78 | 14 | 11 | 11 | 18 | 10 | 11 | 22 | 10 | 12 | 24 | 14 | 11 | 23 | 9 | 14 | [3] 36 | 25 | 20 | ------ | 10 | 13 |
| 19 | 1.12 | 3.66 | 5.70 | 24 | 13 | 9 | 23 | 11 | 9 | 30 | 10 | 8 | 22 | 12 | 10 | 34 | 13 | 9 | [3] 22 | 21 | 18 | [3] 20 | 9 | 8 |
| 20 | 0.98 | 3.57 | 5.50 | ------ | ------ | [3] 5 | ------ | ------ | 5 | ------ | ------ | [3] 4 | ------ | ------ | 4 | ------ | ------ | 5 | ------ | ------ | 9 | ------ | ------ | 12 |

[1] Mils per year average of deepest pits.
[2] Time for De Lavaud cast iron.
[3] Contains one or more actual pit measurements and at least one assigned value which was obtained by dividing 20, the smallest pit measured, by the time in years.
[4] Specimens were not removed.

TABLE 6.—*Comparison of rates of penetration for three periods*—Continued

| Soil No. | Time buried (years) | Bessemer steel (6 specimens) | | | Wrought iron (4 specimens) | | | Pure open-hearth iron (2 specimens) | | | Open-hearth steel (2 specimens) | | | Copper-bearing steel (2 specimens) | | | Sand mold cast iron (2 specimens) | | | De Lavaud cast iron (2 specimens) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third |
| 21 | 1.47 | 4.02 | 6.05 | ---- | 13 | 10 | ---- | 12 | 9 | ---- | 15 | 12 | ---- | 9 | 9 | ---- | 11 | 9 | ---- | 15 | 13 | ---- | 10 | 9 |
| 22 | 1.72 | 3.68 | 5.55 | 27 | 14 | 10 | 21 | 14 | 10 | 25 | 12 | 12 | 22 | 16 | 11 | 27 | 14 | 11 | 32 | 25 | 27 | 18 | 13 | 12 |
| 23 | 1.93 | 4.27 | 6.16 | 26 | 24 | 16 | 25 | 23 | 18 | 28 | 21 | 27 | 45 | 31 | 17 | 34 | 22 | 21 | 60 | 60 | 51 | 18 | 33 | 25 |
| 24 | 1.32 | 4.01 | 6.10 | ---- | ---- | [3] 3 | ---- | ---- | [3] 3 | ---- | ---- | ---- | ---- | ---- | [3] 3 | ---- | ---- | 4 | ---- | ---- | ---- | ---- | ---- | ---- |
| 25 | 1.03 | 3.71 | 5.74 | 23 | [3] 9 | 8 | 20 | 8 | 8 | 23 | 9 | 8 | 26 | 8 | 8 | 24 | 9 | 9 | [3] 29 | 10 | 14 | [3] 21 | 10 | 7 |
| 26 | 1.04 | 3.49 | 5.48 | ---- | 14 | 13 | ---- | 11 | 12 | ---- | 11 | 12 | ---- | 13 | 12 | ---- | 14 | 14 | ---- | 11 | 10 | ---- | 9 | [3] 6 |
| 27 | 1.95 | 4.05 | 5.96 | ---- | [3] 8 | 6 | ---- | [3] 8 | 5 | ---- | [3] 6 | [3] 4 | ---- | 9 | 9 | ---- | 10 | 8 | [3] 16 | 15 | 12 | ---- | (4) | (4) |
| | [2] 2.11 | | | | | | | | | | | | | | | | | | | | | 13 | | |
| 28 | 1.64 | (4) | (4) | 19 | (4) | (4) | 16 | (4) | (4) | 12 | (4) | (4) | 24 | (4) | (4) | 22 | (4) | (4) | 31 | (4) | (4) | 19 | (4) | (4) |
| 29 | 1.96 | 4.07 | 5.99 | 26 | 15 | 15 | 20 | 14 | 11 | 24 | 16 | 17 | 35 | 19 | 13 | 19 | 22 | 14 | 21 | 12 | 19 | 17 | (4) | (4) |
| 30 | 1.11 | 3.65 | 5.69 | [3] 13 | ---- | [3] 4 | [3] 14 | ---- | [3] 4 | 21 | ---- | [3] 4 | 15 | ---- | 4 | 16 | ---- | 5 | ---- | ---- | 5 | ---- | ---- | 5 |
| 31 | 2.01 | 4.07 | 6.02 | 26 | [3] 7 | [3] 6 | 17 | [3] 8 | [3] 4 | 15 | [3] 7 | 4 | 23 | 11 | 9 | 27 | 8 | 8 | [3] 24 | 9 | 10 | ---- | (4) | (4) |
| | [2] 2.07 | | | | | | | | | | | | | | | | | | | | | 20 | | |
| 32 | 0.94 | 3.73 | 5.82 | 25 | 8 | 6 | 29 | 11 | 6 | 43 | 12 | 6 | 24 | 9 | 6 | 29 | 11 | 7 | 36 | 10 | 9 | ---- | 12 | 6 |
| 33 | 1.03 | 3.71 | 5.75 | ---- | [3] 9 | [3] 6 | ---- | [3] 7 | 6 | ---- | 10 | 8 | ---- | 8 | 9 | ---- | 10 | 5 | ---- | 11 | 9 | ---- | 12 | 8 |
| 34 | 1.38 | 4.03 | 6.12 | 15 | [3] 5 | [3] 4 | 16 | [3] 6 | 5 | 14 | [3] 7 | 7 | 20 | [3] 6 | 6 | 21 | [3] 6 | 8 | [3] 19 | ---- | 5 | ---- | 7 | 8 |
| 35 | 1.93 | 4.14 | 6.16 | [3] 11 | [3] 5 | [3] 5 | [3] 11 | [3] 6 | [3] 4 | ---- | [3] 5 | ---- | ---- | [3] 5 | ---- | [3] 13 | [3] 7 | [3] 4 | [3] 17 | 8 | [3] 4 | ---- | (4) | (4) |
| | [2] 2.20 | | | | | | | | | | | | | | | | | | | | | | | |
| 36 | 1.99 | 4.07 | 5.97 | 19 | 9 | 7 | 19 | 9 | 7 | 14 | 9 | 8 | 24 | 12 | 9 | 19 | 10 | 7 | 24 | 12 | 21 | ---- | (4) | (4) |
| | [2] 2.08 | | | | | | | | | | | | | | | | | | | | | 15 | | |
| 37 | 2.00 | 4.07 | 6.03 | 25 | 15 | 9 | 19 | 12 | 7 | 18 | 11 | 8 | 21 | 14 | 8 | 45 | 23 | 10 | ---- | ---- | 9 | ---- | (4) | (4) |
| | [2] 2.07 | | | | | | | | | | | | | | | | | | | | | | | |
| 38 | 1.40 | 4.04 | 6.13 | ---- | ---- | [3] 3 | ---- | [3] 6 | 5 | ---- | ---- | 6 | ---- | ---- | [3] 3 | ---- | ---- | 5 | ---- | ---- | [3] 3 | ---- | ---- | ---- |
| 39 | 1.38 | 4.02 | 6.11 | ---- | 8 | 7 | ---- | [3] 6 | 8 | ---- | 10 | 5 | ---- | 6 | 6 | ---- | 11 | 8 | ---- | 9 | 8 | ---- | 8 | 6 |
| 40 | 1.96 | 4.06 | 5.98 | 17 | 10 | 11 | 15 | 11 | 11 | 23 | 15 | 10 | 20 | 11 | 14 | 22 | 11 | 14 | 16 | 13 | 17 | ---- | (4) | (4) |
| | [2] 2.10 | | | | | | | | | | | | | | | | | | | | | 18 | | |
| 41 | 1.47 | 4.02 | 6.05 | 26 | 11 | 10 | 28 | 10 | 9 | 13 | 8 | 8 | 21 | 9 | 8 | 21 | 9 | 8 | 18 | 10 | 6 | ---- | ---- | 6 |
| 42 | 1.98 | 4.07 | 5.97 | 31 | 19 | 14 | 28 | 19 | 14 | 27 | 15 | 12 | 31 | 20 | 14 | 28 | 18 | 13 | 50 | 38 | 36 | 23 | (4) | (4) |
| 43 | 1.29 | 4.06 | 6.16 | 30 | 13 | 14 | 31 | 18 | 15 | 14 | 9 | 14 | 35 | 19 | 21 | [3] 29 | 14 | 8 | ---- | 24 | 11 | [3] 28 | 14 | 9 |
| 44 | 1.09 | 3.63 | 5.68 | 28 | 14 | 10 | 29 | 12 | 9 | 34 | 21 | 12 | 36 | 12 | 13 | 35 | 14 | 12 | [3] 23 | 19 | 8 | ---- | 12 | 7 |
| 45 | 1.18 | 3.78 | 5.79 | [3] 18 | 7 | 7 | [3] 17 | 8 | 8 | [3] 13 | 9 | 7 | 14 | 9 | 6 | 23 | 12 | 8 | 38 | 14 | 9 | ---- | 12 | 7 |
| 46 | 1.46 | 4.05 | 6.07 | 38 | 22 | 12 | 35 | 16 | 11 | 39 | 20 | 11 | 26 | 14 | 7 | 54 | 27 | 15 | 24 | 13 | 8 | [3] 15 | ---- | [3] 4 |
| 47 | 1.48 | 4.08 | 6.09 | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- | ---- |

[2] Time for De Lavaud cast iron.
[3] Contains one or more actual pit measurements and at least one assigned value which was obtained by dividing 20, the smallest pit measured, by the time in years.
[4] Specimens were not removed.

Table 6 shows data in a few places where the corresponding Table 3 in Technologic Paper No. 368 gives none because in the latter table no figure was recorded if any of the specimens of a class showed no measurable pits. The plan followed in the technologic paper is preferable in that it involves no arbitrary assumptions but it suppresses some data and, if rigidly adhered to, makes it impossible to compute average results for either a soil or a pipe material.

(*b*) Limitations of the Data.—As is indicated at the top of Table 6, the data for some classes of material were obtained from more specimens than were available for some other classes. For example, there were two 1½-inch specimens of Bessemer steel from each of two mills and two 3-inch specimens from a third mill. The outer surface of these specimens upon which pitting data could be obtained amounted to approximately 229 square inches. Only two 1½-inch specimens represented the open-hearth iron, and the exposed area amounted to about 60 square inches. Obviously, the chances for an unusually deep pit caused by abnormal conditions in material or soil were greater for the class of material having the greater exposed area, and the effect of a single deep pit was least when the group has a maximum number of specimens. If each of two groups contains the same number of specimens, the group made up of the larger specimens was at a disadvantage. For example, the cast-iron group, consisting of two 6-inch specimens, exposed about four times the area exposed by the two 1½-inch open-hearth iron specimens. The disadvantage of the larger specimens decreases as the pitting becomes more general, and for most soils it is not very important. Fortunately, we had one material furnished by two manufacturers, each of whom supplied specimens of two sizes. Each manufacturer recognized data on the other's product as representative of his own also. For the first and second periods the average rates of penetration on the 1½-inch specimens of each of these manufacturers was greater than on the corresponding 3-inch specimens by not more than 1 mil per year, while the difference for the third period was 0.6 mil per year in favor of the larger specimens of one maker and zero for the other group. The differences were too small to be of importance. While the average results for all soils was unaffected by the size of the specimen because the total area of the smaller specimens was sufficient to be representative, the same thing may not hold true if materials in one soil are compared, because in the latter case the number of specimens is very limited.

These weaknesses in the pitting data now available are pointed out as one of the reasons why a decision as to the relative merits of materials should be deferred until data on more specimens are available. Later data will also be more accurate because there will be fewer cases in which the depth of the deepest pit will have to be

assumed rather than measured and the conditions to which the specimens have been exposed will be more nearly representative of the average conditions encountered by a pipe line in the course of its life.

Examining Table 6, we find that in 26 soils at least two-thirds of the materials showed a continuous decrease in the rate of pitting while five other soils showed a lower rate in 1928 than in 1926, although the latter rate was greater than in 1924.

It is noticeable that for most of the specimens both the rate of corrosion and the rate of pitting for the 6-year period was nearly the same as for the 4-year period. This indicates that the specimens have reached an approximately stable condition and that in the future the effects of the initial conditions of the test, such as the condition of the pipe surface, the stability of recently disturbed soil, etc., will be of minor importance. It is to be expected, therefore, that future data may be expected to be more nearly representative of the deterioration of pipe lines which have been in the ground a number of years.

TABLE 7.—*Comparison of pitting factors for three periods*

TABLE 7.—*Comparison of pitting factors for three periods*

| Soil No. | Time buried (years) | | | Bessemer steel (6 specimens) | | | Wrought iron (4 specimens) | | | Pure open-hearth iron (2 specimens) | | | Open-hearth steel (2 specimens) | | | Copper-bearing steel (2 specimens) | | | Sand mold cast iron (2 specimens) | | | De Lavaud cast iron (2 specimens) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third |
| 1 | 0.98 | 3.58 | 5.50 | [1] 11.0 | 6.7 | 6.2 | [1] 10.4 | 5.8 | 6.1 | [1] 12.2 | 9.3 | 8.1 | [1] 9.4 | 5.7 | 6.7 | [1] 11.2 | 9.6 | 6.9 | 19.8 | 16.6 | 11.0 | [1] 37.6 | 10.2 | 8.7 |
| 2 | 2.09 | 4.03 | 5.93 | [1] 6.3 | 9.4 | 8.0 | [1] 6.9 | 9.3 | 6.0 | [1] 7.3 | 12.8 | 7.1 | [1] 6.2 | 10.8 | 7.0 | [1] 7.4 | 12.1 | 9.6 | 12.6 | 6.7 | 8.0 | [1] 14.2 | [2] | [2] |
| 3 | [3] 1.94 | | | | | | | | | | | | | | | | | | | | | | | |
| | 2.01 | 4.09 | 6.04 | 16.8 | 13.6 | 12.1 | 19.0 | 12.3 | 10.6 | 27.9 | 16.8 | 14.4 | 17.9 | 10.5 | 14.9 | 22.9 | 15.6 | 10.4 | 20.2 | 9.1 | 18.3 | [1] 10.1 | [2] | [2] |
| 4 | [3] 2.07 | | | | | | | | | | | | | | | | | | | | | | | |
| | 1.38 | 4.03 | 6.12 | [1] 8.8 | 5.0 | 6.6 | [1] 9.8 | 4.7 | 6.8 | [1] 8.7 | 5.9 | 12.2 | [1] 9.1 | 5.7 | 7.9 | [1] 8.1 | 7.2 | 12.0 | 14.5 | 7.1 | 6.2 | [1] 9.0 | 5.2 | 8.0 |
| 5 | 1.93 | 4.13 | 6.16 | [1] 10.7 | 9.0 | 6.8 | [1] 13.1 | [1] 7.2 | 4.9 | [1] 11.6 | 13.1 | 6.0 | [1] 12.8 | 10.0 | 10.4 | [1] 13.3 | 10.2 | 5.7 | [1] 19.5 | 13.3 | 9.5 | 10.0 | [2] | [2] |
| | [3] 2.20 | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 1.93 | 4.13 | 6.17 | [1] 72.5 | [1] 17.6 | 15.8 | [1] 91.5 | [1] 26.6 | 21.4 | [1] 94.2 | [1] 18.4 | 23.6 | [1] 93.2 | [1] 24.8 | 20.2 | [1] 81.5 | [1] 25.2 | 22.0 | [1] 97.6 | [1] 36.7 | [1] 22.0 | [1] 44.0 | [2] | [2] |
| | [3] 2.20 | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | 1.04 | 3.50 | | [1] 20.3 | [1] 8.5 | [2] | [1] 19.6 | [1] 7.2 | [2] | [1] 27.2 | 7.8 | [2] | [1] 24.3 | 7.0 | [2] | [1] 21.1 | [1] 8.9 | [2] | [1] 8.9 | [1] 4.2 | [2] | [1] 18.6 | [1] 4.7 | [2] |
| 8 | 1.13 | 3.80 | 5.83 | 33.2 | 14.4 | 12.0 | 32.7 | 13.4 | 12.7 | 40.2 | 16.2 | 12.8 | 47.1 | 20.3 | 16.5 | 55.3 | 21.5 | 13.9 | 34.3 | 11.6 | 9.5 | [1] 15.3 | [1] 17.0 | 10.9 |
| 9 | 1.03 | 3.48 | 5.47 | 23.2 | 19.2 | 8.0 | 26.8 | 16.3 | 8.2 | 54.0 | 17.9 | 8.0 | 35.5 | 34.5 | 10.9 | 41.2 | 24.3 | 8.3 | [1] 16.1 | 9.6 | 5.9 | [1] 16.0 | [1] 20.6 | 8.8 |
| 10 | 1.33 | 4.01 | 6.10 | 9.7 | [1] 5.2 | 7.8 | 12.2 | [1] 8.5 | 8.0 | 13.1 | [1] 9.7 | 8.8 | 12.9 | [1] 10.9 | 6.8 | 14.1 | [1] 8.4 | 9.4 | [1] 12.4 | [1] 5.4 | 11.9 | [1] 26.6 | [1] 6.8 | 7.4 |
| 11 | 1.36 | 3.97 | 6.01 | 50.3 | 39.0 | 36.7 | 63.3 | 42.1 | 31.0 | 118.4 | 51.3 | 42.8 | 53.4 | 40.8 | 29.5 | 81.0 | 45.4 | 31.4 | [1] 150.0 | 31.1 | 23.1 | 239.0 | 27.0 | 29.4 |
| 12 | 1.94 | 4.14 | 6.17 | [1] 38.4 | 18.7 | 13.8 | [1] 58.2 | 17.2 | 14.8 | [1] 50.7 | 20.2 | 11.1 | [1] 59.3 | 17.7 | 12.7 | [1] 50.2 | 25.8 | 13.6 | [1] 45.1 | 16.4 | 11.8 | [1] 131.0 | [2] | [2] |
| | [3] 2.21 | | | | | | | | | | | | | | | | | | | | | | | |
| 13 | 1.93 | 4.22 | 5.89 | 10.1 | 9.3 | 8.7 | 8.1 | 6.8 | 7.6 | 10.5 | 6.7 | 6.8 | 9.3 | 16.6 | 7.4 | 15.4 | 9.4 | 5.0 | [1] 3.1 | 11.4 | 10.1 | 6.9 | 11.5 | 2.7 |
| 14 | 1.13 | 3.80 | 5.83 | 44.9 | 15.8 | 17.3 | 63.4 | 16.7 | 16.4 | 82.3 | 17.8 | 23.3 | 44.9 | 16.6 | 18.3 | 34.6 | 24.9 | 33.9 | 83.2 | [1] 7.3 | 5.1 | 44.0 | [1] 33.7 | 6.9 |
| 15 | 1.96 | 4.04 | 5.93 | 11.0 | 6.9 | 6.9 | 11.0 | 6.0 | 6.5 | 8.2 | 8.8 | 7.0 | 9.9 | 6.6 | 7.5 | 10.4 | 10.4 | 5.9 | 24.3 | 9.3 | 9.7 | 7.8 | [2] | [2] |
| | [3] 2.08 | | | | | | | | | | | | | | | | | | | | | | | |
| 16 | 1.97 | 4.04 | 5.97 | 10.4 | 11.3 | 14.8 | 10.6 | 8.9 | 10.3 | 14.8 | 10.8 | 11.6 | 9.8 | 11.0 | 9.0 | 14.2 | 13.2 | 12.0 | 11.2 | 14.9 | 10.8 | 19.4 | [2] | [2] |
| | [3] 2.07 | | | | | | | | | | | | | | | | | | | | | | | |
| 17 | 1.21 | 3.83 | 5.87 | [1] 9.0 | 4.7 | 3.4 | [1] 8.5 | 3.9 | 3.1 | [1] 9.2 | 4.4 | 4.2 | [1] 9.0 | 4.6 | 3.5 | [1] 9.2 | [1] 5.3 | 3.7 | 15.6 | 3.9 | 5.9 | [1] 8.7 | [1] 5.2 | 6.0 |
| 18 | 1.19 | 3.74 | 5.78 | 23.0 | 17.9 | 14.8 | 28.5 | 15.3 | 12.8 | 42.1 | 23.2 | 21.0 | 57.8 | 29.8 | 15.7 | 37.7 | 14.6 | 20.1 | [4] | 55.9 | 16.9 | [1] 47.3 | 36.9 | 20.2 |
| 19 | 1.12 | 3.66 | 5.70 | 26.7 | 16.2 | 15.2 | 24.3 | 14.5 | 12.6 | 41.3 | 12.5 | 14.0 | 29.3 | 13.3 | 16.7 | 29.5 | 16.3 | 15.4 | [1] 57.2 | 38.1 | 32.3 | [1] 94.0 | 15.7 | 12.6 |
| 20 | .98 | 3.57 | 5.50 | [1] 12.4 | [1] 6.7 | 8.8 | [1] 14.2 | [1] 6.0 | 7.2 | [1] 13.8 | [1] 6.6 | 5.4 | [1] 12.4 | [1] 6.5 | 6.9 | [1] 13.8 | [1] 7.0 | 8.0 | [1] 10.6 | [1] 5.3 | 7.9 | [1] 15.6 | [1] 6.8 | 12.8 |
| 21 | 1.47 | 4.02 | 6.05 | [1] 6.4 | 13.0 | 8.6 | [1] 6.2 | 12.0 | 7.8 | [1] 6.7 | 17.0 | 11.0 | [1] 6.7 | 8.7 | 7.6 | [1] 6.3 | 11.4 | 8.4 | [1] 5.3 | 11.4 | 8.5 | [1] 7.6 | 9.7 | 7.5 |
| 22 | 1.72 | 3.68 | 5.55 | 18.8 | 9.3 | 8.3 | 16.6 | 9.1 | 7.7 | 21.2 | 9.6 | 8.6 | 17.2 | 10.8 | 8.7 | 17.9 | 9.6 | 10.9 | 36.9 | 13.3 | 20.3 | 17.9 | 12.2 | 11.0 |
| 23 | 1.93 | 4.27 | 6.16 | 4.4 | 5.5 | 4.6 | 4.8 | 5.2 | 4.9 | 6.7 | 5.5 | 8.0 | 8.8 | 7.3 | 4.9 | 5.6 | 6.8 | 4.9 | 4.9 | 5.2 | 5.3 | 3.6 | 6.9 | 5.2 |
| 24 | 1.32 | 4.01 | 6.10 | [1] 47.0 | [1] 29.6 | 16.8 | [1] 56.6 | [1] 26.7 | 19.5 | [1] 66.0 | [1] 30.0 | [1] 14.1 | [1] 49.0 | [1] 36.3 | 13.3 | [1] 65.3 | [1] 40.7 | 19.0 | [1] 80.0 | [1] 32.6 | [1] 13.5 | [1] 67.6 | [1] 58.6 | [1] 12.6 |
| 25 | 1.03 | 3.71 | 5.74 | 37.6 | 23.3 | 16.1 | 33.0 | 20.7 | 16.3 | 35.0 | 33.4 | 20.0 | 48.2 | 21.8 | 19.3 | 39.6 | 22.0 | 22.7 | 117.0 | 13.9 | 23.9 | [1] 52.6 | 19.4 | 17.7 |

[1] Pitting factors calculated assuming total depth of pit to be 0.020 inch.
[2] Specimens not uncovered.
[3] Time of burial of De Lavaud cast iron.
[4] Specimens damaged before weighing.

TABLE 7.—*Comparison of pitting factors for three periods*—Continued

| Soil No. | Time buried (years) | | | Bessemer steel (6 specimens) | | | Wrought iron (4 specimens) | | | Pure open-hearth iron (2 specimens) | | | Open-hearth steel (2 specimens) | | | Copper-bearing steel (2 specimens) | | | Sand mold cast iron (2 specimens) | | | De Lavaud cast iron (2 specimens) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third | First | Second | Third |
| 26 | 1.04 | 3.49 | 5.48 | [1] 15.4 | 16.2 | 30.5 | [1] 14.4 | 12.6 | 28.4 | [1] 16.5 | 15.7 | 31.5 | [1] 16.9 | 15.3 | 37.2 | [1] 14.3 | 17.2 | 38.6 | [1] 10.6 | 7.1 | 15.9 | [1] 14.6 | 10.1 | 16.4 |
| 27 | 1.95 | 4.05 | 5.96 | [1] 25.1 | 5.8 | 7.1 | [1] 20.6 | 6.3 | 5.4 | [1] 33.0 | [1] 7.0 | 7.7 | [1] 22.5 | 17.1 | 11.6 | [1] 27.2 | 9.8 | 9.6 | [1] 6.1 | 12.5 | 11.1 | ------ | [2] | [2] |
| | [3] 2.11 | | | | | | | | | | | | | | | | | | | | | 7.2 | | |
| 28 | 1.64 | ------ | ------ | 6.7 | [2] | [2] | 6.0 | [2] | [2] | 4.0 | [2] | [2] | 8.1 | [2] | [2] | 6.7 | [2] | [2] | 6.4 | [2] | [2] | 4.0 | [2] | [2] |
| 29 | 1.96 | 4.07 | 5.99 | 8.1 | 5.9 | 6.1 | 7.5 | 5.1 | 4.7 | 7.9 | 6.0 | 6.9 | 10.3 | 7.0 | 5.4 | 5.8 | 7.5 | 5.6 | 6.6 | 3.0 | 6.3 | [3] | [2] | [2] |
| 30 | 1.11 | 3.65 | 5.69 | 8.6 | [1] 9.3 | 6.5 | [1] 16.2 | [1] 8.9 | 7.1 | 17.7 | [1] 11.0 | 6.8 | 10.0 | [1] 9.3 | 6.8 | 11.7 | [1] 9.6 | 7.9 | [1] 9.4 | [1] 5.7 | 6.4 | [1] 10.1 | [1] 6.5 | 6.2 |
| 31 | 2.01 | 4.07 | 6.02 | 25.6 | 8.4 | 11.7 | 24.2 | 10.8 | 10.4 | 32.4 | [1] 9.6 | 11.9 | 25.8 | 13.8 | 18.6 | 35.7 | 10.2 | 12.8 | [1] 38.4 | 12.1 | 20.4 | ------ | [2] | [2] |
| | [3] 2.07 | | | | | | | | | | | | | | | | | | | | | 10.7 | | |
| 32 | .94 | 3.73 | 5.82 | 40.9 | 14.0 | 10.3 | 48.9 | 16.5 | 10.3 | 46.1 | 26.1 | 14.0 | 40.8 | 17.7 | 11.5 | 55.8 | 18.9 | 10.8 | 22.4 | 11.6 | 10.7 | [1] 23.7 | 17.3 | 8.6 |
| 33 | 1.03 | 3.71 | 5.75 | [1] 30.2 | 5.6 | 5.2 | [1] 31.2 | [1] 5.0 | 5.1 | [1] 29.8 | 7.3 | 6.5 | [1] 28.1 | 5.8 | 6.2 | [1] 35.5 | 6.7 | 6.1 | [1] 31.7 | 7.6 | 14.7 | [1] 26.4 | 7.7 | 8.2 |
| 34 | 1.38 | 4.03 | 6.12 | 10.9 | [1] 8.6 | 6.8 | 10.8 | [1] 12.5 | 7.5 | 10.0 | [1] 12.5 | 10.2 | 13.7 | [1] 8.1 | 7.8 | 16.1 | [1] 7.4 | 10.8 | [1] 22.1 | [1] 6.5 | 7.2 | [1] 8.9 | 9.5 | 10.0 |
| 35 | 1.93 | 4.14 | 6.16 | [1] 11.7 | [1] 11.0 | 16.9 | [1] 12.8 | 9.4 | 17.7 | [1] 15.7 | [1] 7.6 | [1] 11.0 | [1] 13.9 | [1] 5.6 | [1] 17.8 | [1] 17.8 | [1] 9.9 | 16.3 | [1] 16.3 | 6.8 | 10.9 | ------ | [2] | [2] |
| | [3] 2.20 | | | | | | | | | | | | | | | | | | | | | [1] 10.4 | | |
| 36 | 1.99 | 4.07 | 5.97 | 22.5 | 12.4 | 17.5 | 24.1 | 13.5 | 18.6 | 17.7 | 17.2 | 23.7 | 37.7 | 18.9 | 18.2 | 26.2 | 18.2 | 17.4 | 30.4 | 23.2 | 43.5 | ------ | [2] | [2] |
| | [3] 2.08 | | | | | | | | | | | | | | | | | | | | | 24.8 | | |
| 37 | 2.00 | 4.07 | 6.03 | 13.5 | 9.1 | 8.6 | 10.7 | 7.4 | 6.8 | 10.8 | 7.0 | 7.3 | 11.8 | 8.5 | 7.4 | 26.3 | 12.1 | 10.2 | [1] 7.1 | [1] 2.9 | 5.8 | ------ | [2] | [2] |
| | [3] 2.07 | | | | | | | | | | | | | | | | | | | | | [1] 3.0 | | |
| 38 | 1.40 | 4.04 | 6.13 | [1] 48.0 | [1] 21.8 | 14.9 | [1] 69.0 | [1] 24.5 | 21.2 | [1] 84.0 | [1] 30.0 | 28.8 | [1] 50.6 | [1] 32.6 | 11.8 | [1] 45.6 | [1] 25.1 | 19.1 | [1] 136.5 | [1] 32.6 | 11.4 | [1] 274.0 | [1] 41.9 | [1] 10.3 |
| 39 | 1.38 | 4.02 | 6.11 | [1] 10.5 | 7.2 | 10.1 | [1] 10.5 | [1] 5.7 | 9.3 | [1] 12.3 | 9.2 | 8.8 | [1] 9.2 | 5.2 | 8.0 | [1] 10.1 | 10.3 | 10.1 | [1] 8.0 | 7.5 | 6.3 | [1] 9.5 | 7.5 | 6.6 |
| 40 | 1.96 | 4.06 | 5.98 | 10.8 | 6.4 | 8.2 | 8.6 | 7.1 | 7.5 | 14.1 | 10.2 | 7.5 | 10.5 | 7.1 | 8.4 | 9.9 | 7.2 | 9.7 | 8.3 | 5.5 | 6.7 | ------ | [2] | [2] |
| | [3] 2.10 | | | | | | | | | | | | | | | | | | | | | 10.1 | | |
| 41 | 1.47 | 4.02 | 6.05 | 45.2 | 10.0 | 9.0 | 47.5 | 8.8 | 9.0 | 17.3 | 8.3 | 7.6 | 43.6 | 8.4 | 9.4 | 51.1 | 9.2 | 9.0 | 13.4 | 9.5 | 6.2 | [1] 31.6 | [1] 6.4 | 9.9 |
| 42 | 1.98 | 4.07 | 5.97 | 13.4 | 9.3 | 8.0 | 11.2 | 8.5 | 8.1 | 11.9 | 9.5 | 7.3 | 19.0 | 16.6 | 8.5 | 19.0 | 15.0 | 8.1 | 12.9 | 10.6 | 10.9 | [2] | [2] | [2] |
| 43 | 1.29 | 4.06 | 6.16 | 11.9 | 6.8 | 7.3 | 13.9 | 8.9 | 7.9 | 8.8 | 6.9 | 10.1 | 13.6 | 9.8 | 13.7 | 10.2 | 6.0 | 4.1 | [1] 9.6 | 19.3 | 5.7 | [1] 32.7 | 12.1 | 5.9 |
| 44 | 1.09 | 3.63 | 5.68 | 51.3 | 20.9 | 17.4 | 44.6 | 17.9 | 16.1 | 76.9 | 36.2 | 22.0 | 73.7 | 21.0 | 21.5 | 62.4 | 26.1 | 22.9 | [1] 31.9 | 35.6 | 17.6 | [1] 70.4 | 29.0 | 25.4 |
| 45 | 1.18 | 3.78 | 5.79 | 10.4 | 5.9 | 8.4 | [1] 16.8 | 6.5 | 9.0 | [1] 9.7 | 7.3 | 10.6 | 11.5 | 7.1 | 7.5 | 13.7 | 9.7 | 11.2 | 16.1 | 7.6 | 11.9 | [1] 8.7 | 4.0 | 10.6 |
| 46 | 1.46 | 4.05 | 6.07 | 33.0 | 23.0 | 16.6 | 28.9 | 15.5 | 15.3 | 51.3 | 19.5 | 16.1 | 22.8 | 14.5 | 10.2 | 43.6 | 24.2 | 20.1 | 11.4 | 6.8 | 9.3 | [1] 36.1 | [1] 5.3 | 6.8 |
| 47 | 1.48 | 4.08 | 6.09 | [1] 30.4 | [1] 8.0 | [1] 7.2 | [1] 33.2 | [1] 8.2 | [1] 7.1 | [1] 26.3 | [1] 10.6 | [1] 10.4 | [1] 23.4 | [1] 10.2 | [1] 7.5 | [1] 36.6 | [1] 6.5 | [1] 8.1 | [1] 21.6 | [1] 5.0 | [1] 3.8 | [1] 20.6 | [1] 6.4 | 4.4 |

[1] Pitting factors calculated assuming total depth of pit to be 0.020 inch.
[2] Specimens not uncovered.
[3] Time of burial of De Lavaud cast iron.
[4] These specimens are included with extra specimens.

### 3. PITTING FACTORS

The pitting factor may be defined as the ratio of the depth of the deepest pit to the average depth of corrosion. The latter figure is obtained from the loss of weight, area, and density of the material. The pitting factor is not a measure of the seriousness of corrosion but indicates in a general way the nearness to uniformity of the corrosion. Table 7 shows the average pitting factors for the specimens at the end of three periods. In computing Table 7 the original data on total loss of weight and depth of pits have been used in preference to the average rates of loss and pitting shown in Tables 5 and 6. This somewhat increases the accuracy of the data and explains why, in some cases, the data in Table 7 do not check exactly with results computed from the preceding tables.

It will be noted that, in general, the pitting factor decreased with time. This is one way of indicating that as the specimens grew older the rate of penetration decreased faster than the rate of loss of weight; that is, the corrosion became more generally distributed over the surface of the specimens.

It will be found that, on the average, the pitting factor was smaller for the older and more corroded specimens and that the factors for the 1926 and 1928 specimens were nearly alike and much smaller than for the specimens removed in 1924. The fact has an important bearing on the theory of corrosion. A number of authors have suggested that the presence of corrosion products accelerates corrosion, because the corrosion product is cathodic with respect to the uncorroded metal. From this, the inference has been drawn that a pit once formed tends to deepen at an increased rate because of galvanic action. The theory that the limitation of oxygen at one portion of a surface tends to make that portion anodic also suggests accelerated corrosion at the bottom of pits. The data in Tables 5, 6, and 7 do not substantiate this conclusion but rather tend to show that deep-corrosion product retards corrosion and that, in the later stages, increased corrosion is in the form of spreading more than deepening of the corroded areas, although the latter process continues at a decreasing rate. If later data bear out this conclusion, it will be evident that in determining the future life of a pipe line from the depth of corrosion at the time of inspection it will be incorrect to assume, as is now frequently done, that the depreciation is proportional to the age of the line.

### 4. COMPARISON OF SOILS

(*a*) With Respect to Rates of Corrosion.—Several investigators are attempting to develop methods for determining the corrosiveness of soils by short-time tests. For the trying out of these methods, data on soil corrosiveness as determined from field

conditions are helpful, and the Bureau of Standards data have been used frequently for this purpose. Since rates of corrosion as given for each of the three periods were based on a comparatively few specimens and as the rates for ends of the different periods in some cases varied widely, it sometimes has been difficult to decide the rate of corrosiveness that should be assigned to a soil. It would seem that the most reliable value now obtainable would be based on the total number of specimens involved, on the time of their exposure, and the surface exposed. Table 7 has been prepared on this basis. The values for rates of corrosion were obtained by dividing the sum of all the losses of all the wrought specimens so far removed from a soil by the product of the total area of the specimens removed at one time multiplied by the sum of the periods of burial; that is, approximately 12 years. The result thus obtained was an average rate of corrosion weighted with respect to the size of the specimens and the time they were exposed.

TABLE 8.—*Relative corrosiveness of soils*

| Soil No. | Name | Location | Loss of weight | | | Depth of pits | | |
|---|---|---|---|---|---|---|---|---|
| | | | Ounces per square foot per year | Per cent variation | Rank | Mils per year | Per cent variation | Rank |
| 1 | Allis silt loam | Cleveland, Ohio | 1.02 | 12 | 8 | 12 | 31 | 21 |
| 2 | Bell clay | Dallas, Tex | .75 | 15 | 17 | 9 | 19 | 32 |
| 3 | Cecil clay loam | Atlanta, Ga | .74 | 16 | 18 | 16 | 36 | 5 |
| 4 | Chester loam | Jenkintown, Pa | .83 | 13 | 14 | 10 | 23 | 28 |
| 5 | Dublin clay adobe | Oakland, Calif | .64 | 16 | 21 | 8 | 16 | 36 |
| 6 | Everett gravelly sandy loam | Seattle, Wash | .13 | 23 | 47 | 5 | 46 | 46 |
| 7 | Fairmount silt loam | Cincinnati, Ohio | .70 | 10 | 19 | 10 | 60 | 28 |
| 8 | Fargo clay loam | Fargo, N. Dak | .52 | 10 | 29 | 14 | 66 | 14 |
| 9 | Genesee silt loam | Sidney, Ohio | .50 | 20 | 31 | 13 | 69 | 17 |
| 10 | Gloucester sandy loam | Middleboro, Mass | .55 | 33 | 26 | 8 | 50 | 36 |
| 11 | Hagerstown loam | Baltimore, Md | .24 | 12 | 44 | 16 | 33 | 8 |
| 12 | Hanford fine sandy loam | Los Angeles, Calif | .44 | 30 | 35 | 12 | 14 | 21 |
| 13 | Hanford very fine sandy loam | Bakersfield, Calif | 1.26 | 29 | 5 | 15 | 40 | 11 |
| 14 | Hempstead silt loam | St. Paul, Minn | .54 | 15 | 28 | 20 | 33 | 2 |
| 15 | Houston black clay | San Antonio, Tex | .97 | 14 | 10 | 12 | 33 | 21 |
| 16 | Kalmia fine sandy loam | Mobile, Ala | .87 | 16 | 13 | 16 | 17 | 8 |
| 17 | Keyport loam | Alexandria, Va | 1.01 | 12 | 9 | 7 | 61 | 39 |
| 18 | Knox silt loam | Omaha, Nebr | .45 | 11 | 33 | 13 | 21 | 17 |
| 19 | Lindley silt loam | Des Moines, Iowa | .48 | 21 | 32 | 13 | 46 | 17 |
| 20 | Mahoning silt loam | Cleveland, Ohio | .55 | 36 | 26 | 7 | 76 | 39 |
| 21 | Marshall silt loam | Kansas City, Mo | .82 | 33 | 15 | 12 | 14 | 21 |
| 22 | Memphis silt loam | Memphis, Tenn | .88 | 8 | 12 | 15 | 33 | 11 |
| 23 | Merced silt loam | Buttonwillow, Calif | 2.67 | 16 | 1 | 23 | 16 | 1 |
| 24 | Merrimac gravelly sandy loam | Norwood, Mass | .15 | 27 | 46 | 6 | 72 | 42 |
| 25 | Miami clay loam | Milwaukee, Wis | .29 | 21 | 43 | 11 | 52 | 26 |
| 26 | Miami silt loam | Springfield, Ohio | .42 | 52 | 37 | 14 | 16 | 14 |
| 27 | Miller clay | Bunkie, La | .59 | 34 | 24 | 8 | 16 | 36 |
| 28 | Montezuma clay adobe | San Diego, Calif | 1.93 | 0 | 2 | 20 | 5 | 2 |
| 29 | Muck | New Orleans, La | 1.72 | 10 | 3 | 17 | 24 | 7 |
| 30 | Muscatine silt loam | Davenport, Iowa | .45 | 44 | 33 | 6 | 67 | 42 |
| 31 | Norfolk sand | Jacksonville, Fla | .43 | 28 | 36 | 10 | 60 | 28 |
| 32 | Ontario loam | Rochester, N. Y | .38 | 0 | 39 | 10 | 77 | 28 |
| 33 | Peat | Milwaukee, Wis | .82 | 23 | 15 | 9 | 48 | 32 |
| 34 | Penn silt loam | Norristown, Pa | .52 | 33 | 29 | 7 | 57 | 39 |
| 35 | Ramona loam | Los Angeles, Calif | .31 | 42 | 42 | 6 | 38 | 42 |

TABLE 8.—*Relative corrosiveness of soils*—Continued

| Soil No. | Name | Location | Loss of weight | | | Depth of pits | | |
|---|---|---|---|---|---|---|---|---|
| | | | Ounces per square foot per year | Per cent variation | Rank | Mils per year | Per cent variation | Rank |
| 36 | Ruston sandy loam | Meridian, Miss | 0.36 | 31 | 40 | 11 | 36 | 26 |
| 37 | St. Johns fine sand | Jacksonville, Fla | .91 | 25 | 11 | 14 | 45 | 14 |
| 38 | Sassafras gravelly sandy loam | Camden, N. J | .16 | 6 | 45 | 6 | 55 | 42 |
| 39 | Sassafras silt loam | Wilmington, Del | .63 | 27 | 23 | 9 | 30 | 32 |
| 40 | Sharkey clay | New Orleans, La | 1.03 | 9 | 7 | 13 | 21 | 17 |
| 41 | Summit silt loam | Kansas City, Mo | .64 | 17 | 21 | 12 | 47 | 21 |
| 42 | Susquehanna clay | Meridian, Miss | 1.19 | 9 | 6 | 19 | 23 | 4 |
| 43 | Tidal marsh | Elizabeth, N. J | 1.31 | 11 | 4 | 18 | 32 | 6 |
| 44 | Wabash silt loam | Omaha, Nebr | .39 | 5 | 38 | 15 | 47 | 11 |
| 45 | Unidentified alkali soil | Casper, Wyo | .70 | 33 | 19 | 9 | 44 | 32 |
| 46 | Unidentified sandy loam | Denver, Colo | .58 | 22 | 25 | 19 | 46 | 4 |
| 47 | Unidentified silt loam | Salt Lake City, Utah | .32 | 16 | 41 | 5 | 74 | 46 |

Table 5 shows that in most soils the rate of loss of weight varied from year to year. Probably a small part of this variation was accounted for by differences in the surface conditions or composition of the specimens. In many of the soils a more important cause of the differences was the nonhomogeneity of the soil, which in a 100-foot trench may vary appreciably in texture and moisture. A third possible cause of variations in rates was the variation in rainfall and temperature from year to year. To these possible causes of irregularities in the data must be added the accelerating or inhibiting effects of the corrosion products. All of these things tended to make the data for any one period differ somewhat from those of some other period and must be considered in selecting figures for the relative or absolute corrosiveness of soils. In order to indicate roughly the magnitude of these variations, the departure of the average rate of loss of weight from the weighted average has been indicated in column 5 of Table 8. The value of the average variation was obtained by subtracting from the weighted averaged rate of loss, as given in column 4, the average rate of corrosion of the wrought specimens for each period. The sum of these differences (three in any one soil) was then divided by three times the average weighted rate of loss and multiplied by 100.

The figure thus obtained was not a measure of the accuracy of the data, since part of the deviation was caused by changes in the corrosion phenomena. It is rather an indication of the extent to which the weighted rate of loss obtained at this time is indicative of the final results. In general, the indications are that more data are necessary for rates of loss that are accurately representative of soil corrosivity.

(*b*) WITH RESPECT TO RATES OF PITTING.—Although for a study of corrosion from a chemical standpoint data on rates of loss of weight are probably more significant, many are inclined to judge the seriousness of corrosion by the depth of the pits. For the benefit of

those who wish to study corrosion from this standpoint columns 7, 8, and 9 of Table 8 have been prepared. Column 7 is based on the measurement of the deepest pits on the wrought specimens removed in 1924, 1926, and 1928. Usually two specimens of each material were removed from each soil at the close of each period. This gave a total for each period of eight 1½ inch and eight 3 inch specimens. As there is a probability that a deeper pit will occur on a 3-inch specimen than on a 1½-inch specimen of the same material because of the greater area of the former, it was decided to treat two 1½-inch specimens as equivalent to one 3-inch specimen in weighting the data. The values given for the rates of pitting were, therefore, weighted in accordance with the area of the specimen and the duration of exposure. The following method of determining the weighted average rate of pitting for the wrought materials was used. For the 1½-inch specimens the maximum pit for each material was taken for each period (12 pits); for the 3-inch specimens the maximum pit for each period on each specimen was selected (24 pits). The sum of all these pits (36 pits) was then divided by the number of specimens, usually 36, and by the average time the specimens were buried.

To get the deviation from the mean, the average rate of pitting of the wrought materials for each period was subtracted (algebraically) from the weighted mean average for the three periods. The average difference for the three periods was then obtained. This average difference was then divided by one hundred times the weighted average to get the percentage variation.

Since in most soils there was a tendency for the rate of penetration to decrease with time, the percentage deviation as expressed in Table 8 is not an indication of the precision of the pitting data, but an indication of the tendency of the rate of pitting to change. This tendency to change has, of course, an important bearing on the value of the weighted average rate, as now computed, as an indication of the final rate of pitting, and where the percentage deviation is large we must conclude that the ultimate rate of pitting is correspondingly uncertain. Since the specimens which have been in the ground the longest most nearly represent the condition of old pipe lines, it might be better to use the data in Table 6 rather than that in Table 8 for determining the relative pitting.

(*c*) With Respect to Soil Texture.—The texture of the soil influences the access of oxygen and moisture to buried materials, and in an analysis of the causes of variations in the corrosiveness of soils soil textures should be considered. In doing so the student should bear in mind the fact that the system of soil nomenclature which has been used to designate soils was developed for use in connection with agricultural problems in which the surface layers play the most important part. On this account the texture of the

soil as indicated in the soil name is that of the first 18 inches of soil and may or may not be that of the subsoil in direct contact with the pipe specimens in the Bureau of Standards tests. Those wishing information as to the texture of the soil adjacent to the pipe should consult the soil descriptions given in Technologic Paper No. 368 of the Bureau of Standards. An idea of the soil texture adjacent to the pipe can also be obtained from Table 3, since there is a fairly close relation between soil texture and the amount of colloids.[5]

### 5. COMPARISON OF MATERIALS

Of primary importance to many readers of the reports on soil corrosion is the question of the relative merits of the materials under test. We wish to point out that the object of the investigation is to study the relation of soils to corrosion of pipe lines rather than to determine the relative merits of different kinds of pipes and that the data required for the latter purpose must necessarily be much more extensive than for the former. At the end of each test period 24 samples of pipe were removed from each location, but of these only two were of one material and manufacture. Frequently the data on the two specimens of the same material differed as widely as those for different materials. This may be accounted for by the unavoidable differences in individual specimens of almost any commercial pipe material or by minor differences in the composition, texture, or moisture of the soil in different parts of the same trench. It is expected that as the experiment runs for longer periods and more specimens of each material are removed for examination these accidental differences will counteract each other and the average results will become fairly representative of the characteristics of the several materials tested. At this time but half of the number of the bare pipe specimens have been removed. By weighting the data in accordance with the time each set of specimens was exposed before removal, we find that the weight to be assigned to the data obtained to date is to that at the end of the test as 12 is to 42. Evidently, therefore, the dependability of the final results will be much greater than any that are now available. The reliability of individual figures improves as the specimens are exposed longer because the percentage error in weighing and measuring becomes less. Trench conditions are becoming more stable and more representative of the conditions to which pipes are exposed. There appear to be progressive changes in the rates of corrosion and pitting, and these changes in rates seem to differ for different materials. On account of all of these things which may affect the final results of the test, the Bureau of Standards believes that it is unwise at this time to draw conclusions as to the relative merits of the materials under test, and the data are published

[5] See Methods of Classifying Soils, B. S. Tech. Paper No. 368, p. 459.

from time to time prior to the conclusion of the investigation only in order that those cooperating with the bureau in the study of corrosion may have promptly all the data they have helped to obtain. It is earnestly hoped that all those who study these data will draw a sharp distinction between the actual facts presented and conjectures reached through attempts to interpret the relation between the now limited data and some specific corrosion problem.

### 6. APPLICATION OF RESULTS TO SPECIFIC CORROSION PROBLEMS

A number of requests have been received for data on the corrosiveness of the soil in certain cities and the best material for use there. Such requests show progress in that they indicate the recognition that different soils may call for different pipe materials, but unfortunately the area within a city usually includes a number of widely different soils, and the selection of the best pipe not only requires more data on pipe materials than are now available but a soil survey of the route of the pipe line and a correlation of the soil data with corrosion. The data for such a correlation are also inadequate at this time, although the problem is being studied at the Bureau of Standards and by a number of other organizations. It appears probable that a method for roughly estimating the corrosiveness of a soil in any locality will be available within a few years.

## V. RESULTS OF SUPPLEMENTARY RESEARCH ON PIPE LINES

The early results of the soil-corrosion investigation made it apparent that the work already under way would not solve some of the most pressing corrosion problems and that additional tests would be required to explain the causes of corrosion, identify corrosive soils, and suggest means of mitigating corrosion. Some of this work has been undertaken during the past year. The results are insufficient to be conclusive but are worthy of note because of the things which they suggest.

### 1. INSPECTION OF GAS MAINS

However planned, laboratory or field experiments seldom, if ever, completely duplicate actual operating conditions. For this reason the cooperation of a number of gas companies in the study of the condition of their cast-iron pipe lines was secured. The purpose of the undertaking was to determine the extent of corrosion on lines passing through a few of the soils in which the bureau's specimens were buried and to make a comparison between the corrosion of these lines and that of the specimens.

Attempts to find suitable places for this comparison made it evident that a large portion of the gas mains in city streets pass

through earth which has been so modified by city conditions, such as grading, filling, and artificial drainage, that it does not fit into the recognized soil classification, and the data on such soils are not entirely suitable for comparison with those on the selected soils in which the field specimens were buried. Pipes were examined in 40 locations representing Cecil, Memphis, Muck, Norfolk and Susquehanna soils. In Cecil and Norfolk soils the conditions of the pipes checked quite well with the experimental data. In Muck, where there was a possibility, though little probability, of electrolysis, the pipes were more seriously corroded than was expected, while in Memphis silt loam the pipes, most of which were much older than the test specimens, showed a lower rate of pitting than was anticipated. Susquehanna soils gave inconsistent data, probably because this soil group represents soils in the process of change, and, therefore, soils which differ chemically although classified as the same soil. In the case of the other soils studied the same soil in different localities gave fairly consistent indications as to corrosive properties. On the whole, the results of the examinations were somewhat unsatisfactory as checks on the experimental data. In one respect, however, the data were consistent. In each soil, comparison of the depth of pits on pipes of different ages showed that the rate at which pits penetrate the pipe decreases with time. This is in accord with the experimental data and indicates that a straight line depreciation curve should not be used for the determination of the life of an underground pipe system.

### 2. CORROSION ACCOMPANIED BY GALVANIC ACTION

In the summer of 1927 the Bureau of Standards cooperated with a midcontinent oil company in an attempt to determine the relation between observed corrosion on a line which was being removed and soil conditions. It was observed that where the line passed through certain geological strata corrosion was severe while it was a minimum in one other stratum. Measurements of differences of potential along single pipe lengths indicated electrical currents which were different at different points on the pipe, but which were steady and changed little from day to day. Figure 1 shows the currents in one section of the line. The direction of flow of the current on the pipe was different at different points, although at a given point the current was steady. Considerably more corrosion was observed on lengths of pipe which were discharging current than on lengths collecting current. The conclusion was reached that the passage of the pipe line through different soils resulted in the formation of a galvanic cell, the electrodes of which were in some cases a mile or more apart.

In the case referred to, the maximum current on a 4-inch steel line was slightly more than 1 ampere. A similar study was later made on an 8-inch steel line in California in which a similar relation between

corrosion and current discharge was observed and a maximum current of 4.6 amperes found. If, as seems probable, these galvanic cur-

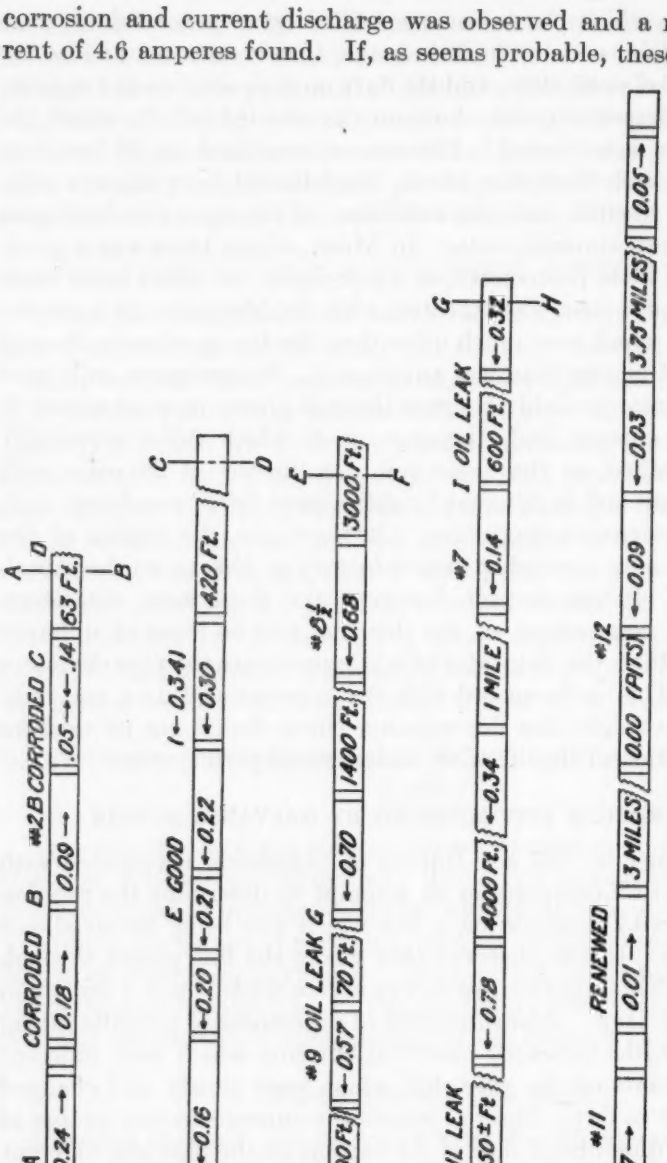


FIGURE 1.—*Current readings on 4-inch steel line*
Arrows indicate direction of flow of current.

rents accompany pipe-line corrosion, it would indicate that corrosion on short isolated sections of pipe should be different from that on a continuous line of the same material under similar soil conditions and

that a welded or screw-jointed line will corrode more rapidly in some locations than one with numerous insulating joints. It would indicate also that a protective coating should be an electrical insulator and that coating the pipe at points of no corrosion assists in reducing the corroding action at other sections. The question has a bearing on the proper specifications for protective coatings and on the method of their application.

TABLE 9.—*Corrosion of high-silicon cast iron and lead sheath*

| Soil No. | | | Corrosion loss (ounces per square foot per year) | | | Rates of pitting (mils per year) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | A | | H | |
| Bureau of Standards report | American Foundrymen's Association report | Time buried (years) | D | A | H | Maximum | Average, 5 deepest | Maximum | Average, 5 deepest |
| 1 | 3 | 5.50 | 0.006 | 0.27 | 0.25 | 3 holes | | 20 | 15 |
| 4 | 7 | 6.12 | .008 | .47 | .17 | 8 | 8 | 8 | 7 |
| 5 | 9 | 6.16 | .068 | | | | | | |
| 6 | 10 | 6.17 | .003 | | | | | | |
| 8 | 12 | 5.83 | .001 | .09 | .03 | N | | S | |
| 9 | 13 | 5.47 | .021 | .18 | .10 | S | | 6 | 5 |
| 10 | 14 | 6.10 | .002 | .07 | .06 | S | | S | |
| 11 | 15 | 6.01 | .002 | .07 | .05 | 6 | | S | |
| 13 | 5 | 4.89 | .001 | | | | | | |
| 14 | 17 | 5.83 | .051 | .20 | .12 | 8 | 7 | 7 | 5 |
| 17 | 19 | 5.87 | .019 | .11 | .05 | 8 | 6 | 4 | |
| 18 | 20 | 5.78 | .005 | .26 | .04 | 9 | 7 | 8 | 5 |
| 19 | 21 | 5.70 | .077 | .16 | .08 | 5 | 4 | S | |
| 20 | 22 | 5.50 | .039 | .42 | .19 | 13 | 11 | 8 | 7 |
| 21 | 23 | 6.05 | .000 | .29 | .17 | 7 | 7 | 5 | 4 |
| 22 | 24 | 5.55 | .004 | | | | | | |
| 23 | 47 | 6.16 | .007 | .06 | .03 | 4 | | N | |
| 24 | 25 | 6.10 | .007 | | | | | | |
| 25 | 26 | 5.74 | .141 | .09 | .06 | S | | N | |
| 26 | 27 | 5.48 | .000 | .08 | .04 | S | | 6 | |
| 27 | 45 | 5.96 | .155 | | | | | | |
| 29 | 28 | 5.99 | .035 | | | | | | |
| 30 | 29 | 5.69 | .096 | .10 | .08 | 5 | 4 | 6 | 4 |
| 32 | 32 | 5.82 | .011 | .09 | .06 | N | | 4 | |
| 33 | 33 | 5.75 | .078 | .11 | .12 | S | | S | |
| 34 | 34 | 6.12 | .032 | .61 | .22 | 8 | 7 | 7 | |
| 36 | 36 | 5.97 | .002 | | | | | | |
| 38 | 40 | 6.13 | .006 | .26 | .04 | G | | 6 | |
| 39 | 41 | 6.11 | .008 | .18 | .07 | 5 | 4 | 4 | |
| 40 | 8 | 5.98 | .006 | | | | | | |
| 41 | 43 | 6.05 | .003 | .07 | .06 | S | | 5 | |
| 42 | 44 | 5.97 | .003 | | | | | | |
| 43 | 38 | 6.16 | .441 | .04 | .02 | N | | S | |
| 44 | 46 | 5.68 | .056 | .18 | .07 | 5 | | 5 | |
| 45 | 1 | 5.79 | .051 | .05 | .04 | 10 | 7 | 9 | 8 |
| 46 | 39 | 6.07 | .007 | .13 | .02 | 5 | 4 | 5 | 4 |
| 47 | 42 | 6.09 | .067 | .25 | .05 | 1 hole | | S | |

A, Antimony lead.
D, high silicon cast iron.
H, commercially pure iron.
N, no pitting.
S, few pits less than 0.020 inch.
G, general pitting less than 0.020 inch.
Dimensions of exposed areas of specimens:
A, 20.5 by 8.5 inches by 0.130 inch.
H, 20.5 by 3.5 inches by 0.112 inch.
D, ID, 3 inches. OD, 3.625 inches. Length, 6 inches. Area, 131.52 square inches.

## VI. RESULTS OF TESTS OF OTHER MATERIALS USED UNDER GROUND

### 1. CORROSION OF HIGH-SILICON CAST IRON

There were included in the materials removed in 1928 specimens of cast iron containing approximately 14 per cent of silicon. While these specimens were in the form of short sections of pipe, and while the material is used for plumbing in chemical plants, its physical properties are so different from those of the other pipe materials under test that it seems best to report on it separately. Table 9 gives the average rate of loss of weight for the specimens removed in 1928. Two specimens were removed from each soil. No pits of appreciable depths were observed, and it will be noted that the losses in weight were very small. In many of the soils most of the surface of the specimens appeared bright after six years.

### 2. CORROSION OF LEAD CABLE SHEATH

In 1922 specimens of lead cable sheath, such as is used for telephone and power cables, were buried in 27 soils; additional specimens were buried in other soils two years later. Before burial the sheaths were slit and opened out to form sheets. The specimens of telephone sheath were 0.13 inch in thickness and had a total exposed area of approximately 174 square inches. This material contained a small percentage of antimony. The other specimens of lead sheath were of commercially pure lead 0.112 inch in thickness and had an exposed area of approximately 72 square inches. The large specimens were designated "A" in Table 9, while the smaller specimens were designated by the letter "H."

Lead is so soft that it is easily scratched, and on this account it is probable that a small part of the loss recorded in Table 9 was due to handling rather than to corrosion. It will be noted that in most soils the corrosion losses and rates of pitting were somewhat greater for the sheath containing antimony. Specimens removed previously gave similar results.

These specimens were buried on edge directly in contact with the soil. The conditions to which they were exposed were, therefore, somewhat different from those encountered by cable sheaths in duct lines. Although cables in ducts are frequently partially in contact with mud and water, they are also exposed to air. It is impossible to say which condition is worse.

Occasionally lead-sheathed cables are buried directly in the earth. The data in Table 9 indicate that this is a questionable practice, since we found the antimony lead sheath punctured within six years in two soils and pits at least one-third of the thickness of the sheath in nine other soils at the end of the 6-year period.

### 3. CORROSION OF PARKWAY CABLE

For burial directly in the earth a special cable has been designed. This consists of one or more rubber-covered copper conductors incased in lead sheath which is protected by a layer of jute impregnated with oil or bitumen followed by two layers of galvanized-steel tape wound spirally in opposite directions and an outer covering of bitumen-impregnated jute. Samples of this material were buried along with the samples of cable sheath. Table 10 indicates in a general way the condition of the specimens removed in 1928.

TABLE 10.—*Condition of parkway cable*

| Soil No. | Time buried (years) | Outer fabric | Inner fabric | Outer steel tape | Inner steel tape | Lead sheath |
|---|---|---|---|---|---|---|
| 1 | 5.50 | G | G | R | SR | W |
| 4 | 6.12 | B | G | SR | SR | TW |
| 8 | 5.83 | F | G | G | G | TW |
| 9 | 5.47 | F | G | SR | VSR | TW |
| 10 | 6.10 | B | G | G | G | TW |
| 11 | 6.01 | B | G | G | G | G |
| 13 | 5.89 | B | G | BR | SR | W |
| 13 | 5.89 | F | G | G | SR | W |
| 13 | 5.89 | F | G | G | SR | TW |
| 17 | 5.87 | G | G | G | G | G |
| 18 | 5.78 | B | G | SR | G | W |
| 19 | 5.70 | F | G | VSR | G | TW |
| 20 | 5.50 | B | G | VSR | G | TW |
| 21 | 6.05 | F | G | VSR | G | W |
| 23 | 6.16 | B | F | Failed | BR | W |
| 26 | 5.48 | B | G | G | G | TW |
| 30 | 5.69 | F | G | R | SR | W |
| 32 | 5.82 | B | G | R | SR | W |
| 34 | 6.12 | B | G | G | G | TW |
| 38 | 6.13 | B | G | VSR | G | TW |
| 41 | 6.05 | B | G | SR | G | W |
| 43 | 6.16 | F | G | R | VSR | G |
| 45 | 5.79 | F | G | VSR | G | G |
| 46 | 6.07 | B | G | BR | SR | TW |
| 47 | 6.09 | B | G | G | G | W |

Rating symbols:
B, bad.
G, good.
F, fair.
R, rusted.
SR, slightly rusted.
VSR, very slightly rusted.
BR, badly rusted.
TW, thin white corrosion product on lead sheath.
W, white corrosions product on lead sheath.

## VII. SUMMARY

The specimens removed in 1924, 1926, and 1928, constitute half of the specimens buried in 1922, but on account of the longer time involved in the data on specimens to be removed later, the relative weight to be given data now available is less than a third that which should be assigned to the results at the end of the investigation. Through the cooperation of private organizations additional information on the properties of soils and materials are now available.

Supplementary field investigations have corroborated the bureau's results in their indication that the rates of penetration of pits in cast iron decrease with the age of the pipe. Limited studies of oil lines

have indicated that part of the corrosion found is the result of the formation of a galvanic cell when the pipe line passes through two different soils. This has a bearing on the type of joint and the kind of coating to be used in such lines.

Considerable progress has been made in the laboratory study of corrosion.

Tables have been prepared showing the corrosion data for the specimens removed in 1924, 1926, and 1928. Data for the last two periods are, in most cases, similar and indicate that the conditions of test have reached a nearly steady state. There are no data indicating that the tentative conclusions drawn in earlier reports should be changed.

On the average the rate of corrosion was decreasing, although this was not universally true. The rate of pitting decreased in most soils, and increased corrosion seems to be largely the result of the spreading of the corrosion, penetration progressing more slowly than loss of weight.

Slight differences in the composition of different specimens of the same material, occasional flaws, and especially the variations in the soil at different points in the same trench result in variations in data which make it impossible to estimate the significance of small differences in apparent rates of corrosion until more specimens have been examined. It is therefore urged that no conclusions as to the relative merits of the materials or as to the ultimate life of any material be drawn until more data are available.

WASHINGTON, March 18, 1929

# PHOTOIONIZATION OF SOME ALKALI VAPORS

By F. L. Mohler and C. Boeckner


## ABSTRACT

Relative measurements of photosensitivity as a function of wave length have been made by the space-charge method, and absolute values at favorable wave lengths have been measured by a direct method.

*Caesium.*—The shape of the sensitivity curve is similar to published results and independent of pressure. The photoionization is proportional to the pressure giving an atomic absorption coefficient $k$ of $2.3\times10^{-19}$ at the principal series limit.

*Rubidium.*—The sensitivity curve is similar to caesium and the value of $k$ is $1.1\times10^{-19}$ at the limit.

*Potassium.*—The sensitivity is low at the limit 2856 A and rises rapidly from 2600 to 2200 A.

*Absorption of caesium.*—A rough measurement at low pressure gave a value of $3\times10^{-19}$ at 3100 A, but with an uncertainty of the same magnitude as the effect measured.

*Recombination of caesium.*—An estimate of the value of $k$ for caesium, based on measurements of the intensity of recombination radiation, gives the same magnitude as the photoionization value.


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## I. INTRODUCTION

The absorption of alkali vapors for radiation of a frequency greater than the principal series limits and the consequent ionization of the atoms has been studied by a variety of methods with rather contradictory and puzzling results. Careful measurements of the absorption of sodium and potassium vapor have been made by Harrison [1] and Ditchburn [2] and Trumpy.[3] Harrison found that the continuous

[1] Harrison, Phys. Rev., **24**, p. 466; 1924.
[2] Ditchburn, Proc. Roy. Soc., **117**, p. 486; 1928.
[3] Trumpy, Zeits. f. Phys., **47**, p. 804; 1928.

absorption was a maximum near the sodium limit and dropped gradually toward the violet, but he concluded from other results that a large and unknown part of this absorption came from molecular aggregates, presumably $Na_2$. Trumpy's absorption curves show a sharper maximum and are interpreted to be of atomic origin. Ditchburn attempted to separate molecular and atomic absorption in potassium from analysis of measurements at different pressures, using the line absorption as a measure of the atomic concentration, but he found that the form of the resulting curves depended greatly on the pressure. In some cases it dropped and then rose in going from the limit toward the violet while in other cases there was a continual rise. He estimated the atomic absorption coefficient at the limit as $10^{-19}$.

Measurements of the ionization produced by radiation offer the advantage that much lower pressures can be used. Lawrence [4] and Williamson [5] have studied the photoionization of potassium by illuminating a jet of vapor. The former found that radiation on the red side of 2600 A was ineffective, while beyond this point the efficiency increased rapidly. Williamson found a small effect beginning near 3100 A in addition to the increase beyond 2600 A. The limit is at 2856 A. Figure 5 includes these results.

Measurements in caesium have given entirely different results. Mohler, Foote, and Chenault [6] have used the effect of positive ions on a thermionic current limited by negative space charge to detect photoionization. This method and the vapor-jet method do not yield absolute values. Subsequently, Little [7] has made direct measurements in vapor under equilibrium conditions. Both results show a sharp maximum accurately at the limit with a minimum near 2700 A and a gradual increase beyond this point. (See Fig. 2.) The last feature has been questioned in a recent note.[8] Little gives the atomic absorption coefficient as $2.5 \times 10^{-21}$ at the limit which is surprisingly low.

In a recent paper [9] the authors derived the relative probability of recombination of caesium ions and electrons as a function of the electron speed from the intensity distribution in the continuous emission band beyond the subordinate series limit. From this, by the principal of detailed balance, it was computed that the continuous absorption of atoms in the 2 $P$ state varied as $\lambda^4$, a decrease much slower than that shown by the photoionization measurements for the 1 $S$ state.

These results are in marked contrast with absorption measurements in the X-ray range where absorption always begins at the limit and

[4] Lawrence, Phil. Mag., **50**, p. 345; 1925.
[5] Williamson, Proc. Nat. Acad., **14**, p. 793; 1928.
[6] Mohler, Foote, and Chenault, Phys. Rev., **27**, p. 37; 1926.
[7] Little, Phys. Rev., **30**, p. 109; 1927; and correction note, p. 963.
[8] Lawrence and Edlefson, Phys. Rev., **33**, p. 265; 1929.
[9] Mohler and Boeckner, B. S. Jour. Research, **2** (R. P. 46), p. 489; 1929.

decreases as $\lambda^3$. (The photoionization would vary as $\lambda^4$.) In view of these discrepancies, it seemed worth while to make absolute and relative measurements of photoionization for several of the alkali vapors using the same procedure for each. This paper includes measurements with caesium, rubidium, and potassium, though only relative measurements were made in the last case. The absolute measurements in caesium vapor were made with the cooperation of W. W. Coblentz and R. Stair and have been described in a joint note.[10] The material has been included in the present paper for completeness.

## II. METHOD AND APPARATUS

From the definition of the atomic absorption coefficient $k(\nu)$ the amount of energy $E$ absorbed from a beam of energy flux $J(\nu)$ in a length $l$ is

$$E = J(\nu)\ N\ p\ l\ k(\nu)$$

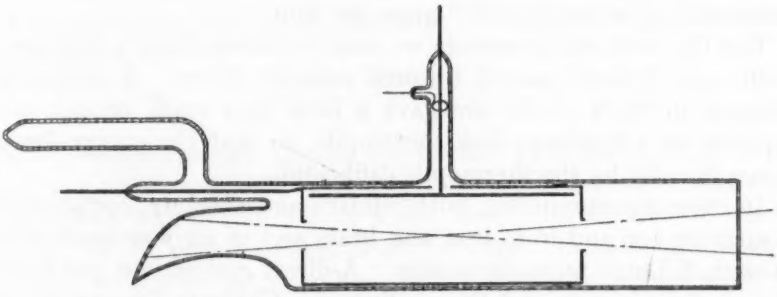

FIGURE 1.—*Tube for direct measurement of photoionization*

where $N$ is the number of atoms per cm$^3$ at unit pressure and $p$ is the pressure. The number of ions produced per second $I/e$ (current/electron charge) will equal the number of quanta absorbed $E/h\nu$, so that

$$k(\nu) = I\ h\ \nu/e\ N\ p\ l\ J(\nu) \qquad (1)$$

The sensitivity $I/J(\nu)$ is thus proportional to $k(\nu)/h\nu$ or to Einstein's probability of absorption, $B$.

The determination of $k(\nu)$ is conveniently made in two parts—a measurement of the relative sensitivity at different wave lengths and an absolute measurement of $k$ at a favorable wave length. For the first purpose the space-charge method was selected because of its high sensitivity and the entire absence of surface effects. Absolute measurements were made under conditions such that the radiation flux could be definitely determined. It was not essential to use monochromatic radiation provided the spectral intensity distribution was known.

[10] Mohler, Boeckner, Stair, and Coblentz, Sci., 69, p. 479; 1929.

Figure 1 illustrates the ionization chamber used for the direct measurement of the photoionization. One electrode is a cylinder 8 cm long and 3 cm in diameter with holes 1 cm in diameter at each end. The other is a plate nearly as long as the cylinder and 1 cm wide. Electrodes were platinum and the containing tube quartz. The beam of light focussed near the center of the cylinder was small enough, so that it passed through the holes without striking either. The cylinder was made positive and the plate negative to further reduce the possibility of photo-electric emission from electrode surfaces. Guard rings prevented any surface leak to the plate, but at an operating temperature of 220° C. there is a thermionic emission of the order of $10^{-9}$ amp. Under steady temperature conditions this gave no trouble. Applied voltages of from 2 to 10 volts were required to secure a saturation ion current, and frequent check measurements were made to be certain that the operating voltage was adequate. Currents were measured by a Leeds & Northrup galvanometer at a sensitivity of about $5 \times 10^{-11}$ amp. per mm.

For the work with caesium we used radiation from a mercury arc (110 volt Uviarc) passed through suitable filters. A circular diaphragm in front of the arc gave a focal spot small enough to fall entirely on a Coblentz disk thermopile, so that the energy flux was given directly by the thermopile calibration.

In other measurements, both relative and absolute, radiation from a mercury arc and from iron and brass arcs in air was resolved by a Bausch & Lomb monochromator. A direct comparison was made of the photoionization and the reading of a Coblentz linear thermopile at the exit slit of the monochromator. This was corrected by later observing the thermopile reading at the position of the tube. Slit widths ranging from 20 A to 60 A at 3100 A were used. With the 60 A slit the 3130 lines of the mercury arc gave a radiation flux of about 100 ergs $sec^{-1}$ while an 8-amp. iron arc gave three to seven times this energy at 2750, 2550, and 2350 A. The brass arc gave a comparable intensity near 2200 A. Thermopiles and galvanometer were calibrated from time to time under operating conditions by means of a radiation standard.[11]

Measurements of the relative sensitivity at different wave lengths were made by the space-charge method following the procedure described by Foote and Mohler.[12] A hairpin cathode near the axis of a cylindrical anode is operated so that the current is limited by the space charge to a small fraction of the saturation current. The presence of positive ions increases the current; and, by balancing out the dark current, changes of $10^{-9}$ amp. can be measured. With caesium in the tube the radiation gave a current change of the order

[11] Coblentz, B. S. Sci. Paper No. 227.
[12] Foote and Mohler, Phys. Rev., **26**, p. 195; 1925.

of $10^{-5}$ amp., and when the effect is as large as this the current ceases to be proportional to the intensity. It was necessary to determine the current-intensity relation by measuring the effect produced when a series of wire gauze screens of known transmission were placed in the light beam.

Tubes were sealed off from the pumps after distilling in the alkali metal and thoroughly outgassing. In operation, the body of the tube was held at a fairly steady temperature of about 220° C. while a small side tube containing the alkali metal was held at a lower temperature by a separate heater. This reduces the error in determining the vapor pressure; but even so, uncertainties in vapor-pressure data and in our temperature measurements make $p$ the most uncertain quantity in equation (1).

## III. RESULTS

### 1. CAESIUM

Vapor pressures were computed from an equation, based on values by Kroner;[13]

$$\log p = -\frac{3966}{T} + 7.1650$$

Relative measurements of the sensitivity were made with monochromatic radiations from mercury, iron, and brass arcs. The mercury lines on the red side of 3130 A give no effect while the 3130 group gives by far the greatest effect of all the lines. Because of the sensitivity of the space-charge method the precision was limited solely by the thermopile measurements and the inherent unsteadiness of air arcs Figure 2 gives these results and, for comparison, the published curves of Little [14] and of Mohler, Foote, and Chenault.[15] Repeated measurements in the range 2750 to 2200 A confirm the upward trend of the sensitivity curve at the short wave length end. Measurements with caesium at 180° C. (vapor pressure 0.03 mm) and at 115° C. (0.001 mm) gave curves of identical shape.

The best absolute measurements were obtained with radiation from the mercury arc passed through a water cell and Corning purple glass G986 A. The transmitted light contains the lines 4047, 3650, 3300, 3130, 3010, and 2967 A. Addition of a piece of ordinary window glass removes 3130 and the lines of shorter wave length. It reduces the photoionization to zero and the thermopile readings to about 50 per cent. From energy measurements with the monochromator and the curve of Figure 2 we found that about 31 per cent of the radiation transmitted by the purple glass and 78 per cent of the photoionization

[13] Rowe, Phil. Mag., **3**, p. 534; 1927. (A critical summary of vapor tension data for all alkalies.)
[14] See footnote 7, p. 304.
[15] See footnote 6, p. 304

came from 3130 A. Corrections for absorption in the tube and furnace windows were made. Figure 3 is a plot of the photoionization as a function of the vapor pressure over the temperature range 146° to 201° C. The effect is proportional to the pressure within experimental error. This fact and the absence of any effect for radiation on the red side of 3130 support the conclusion that surface photoelectric effect has been eliminated.

The caesium used in most of the work did not have a sharp melting point. The experiment was repeated with a carefully prepared sample which melted sharply near 28° C. and one run gave the same value as

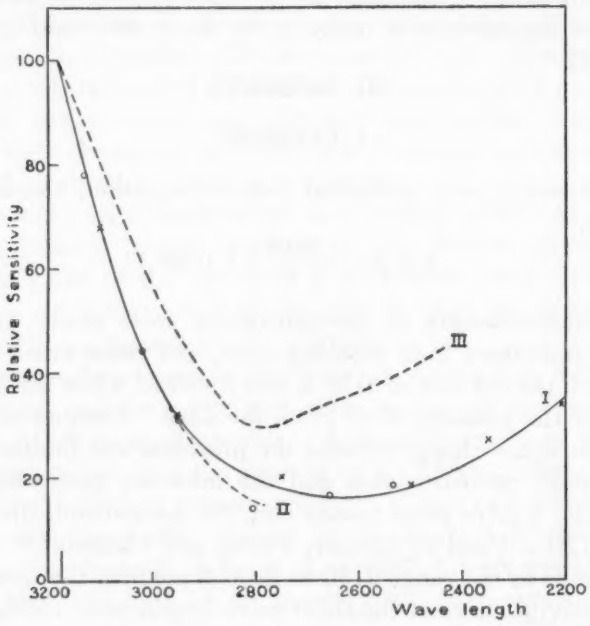


FIGURE 2.—*Photo-electric sensitivity of caesium vapor*

*I*, circles are based on measurements with a mercury arc, crosses on radiation from iron and brass arcs. *II*, is from published data of Mohler, Foote, and Chenault; *III*, from data of Little

before. Then, apparently a small air leak developed, and somewhat lower values were obtained though the oxidation of the caesium was barely visible. On the basis of the slope of the line in Figure 3 the value of $k$ at 3130 is $1.85 \times 10^{-19}$. Measurements with a filter of silvered quartz which transmitted only the 3130 group and with the monochromator checked this value but with somewhat greater experimental error. From the value of $k$ at 3130 and the sensitivity curve of Figure 2 we conclude that $k = 2.3 \pm .2 \times 10^{-19}$ at the limit 3184 A.

For the first time we are able to give a figure for the efficiency of the space-charge method. In the tube used in these experiments a

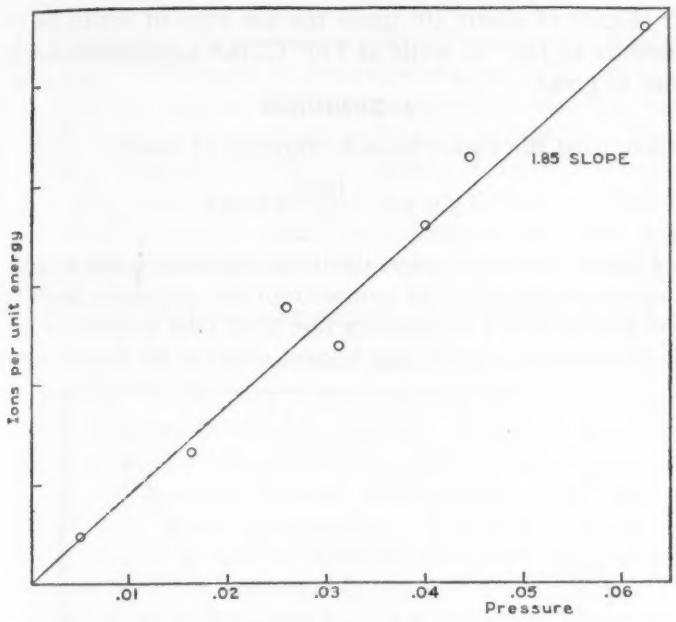


FIGURE 3.—*Photoionization in caesium vapor as a function of the pressure*

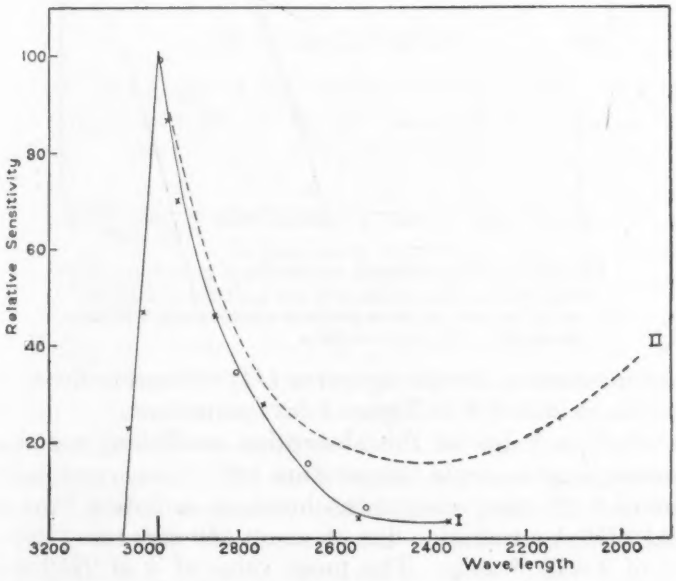


FIGURE 4.—*I Photo-electric sensitivity of rubidium vapor*

Circles are from measurements with a mercury arc, crosses from measurements with an iron arc. The limit is at 2968 A. *II* is Curve *I* for caesium with the wave-length scale shifted to bring the limits together

current change of about $10^4$ times the ion current could be obtained with caesium at 180° C. while at 110° C. the amplification was about ten times as great.

### 2. RUBIDIUM

We have used the vapor tension equation of Scott,[13]

$$\log p = -\frac{4209}{T} + 7.331$$

Figure 4 shows the relative sensitivity at different wave lengths based on the space charge effect of monochromatic radiation from the mercury and iron arcs. The mercury line 2967 falls within an Angstrom of the limit and gives by far the biggest effect of all the mercury lines.

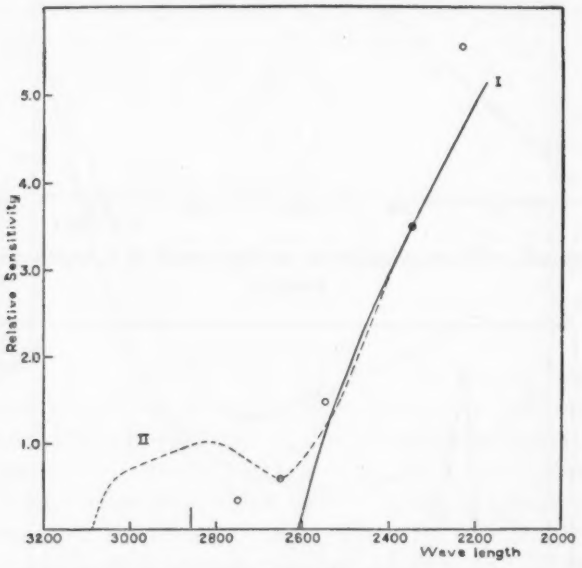


FIGURE 5.—*Photo-electric sensitivity of potassium vapor*
Circles give our measurements with iron and brass arc radiation. *I* and *II* are from published results by Lawrence and Williamson, respectively. The limit is at 2856 A

A plot of the caesium sensitivity curve (*II*) reduced to the same wave length scale is included in Figure 4 for comparison.

The absolute value of the absorption coefficient was based on measurements at a single temperature 196° C., corresponding to a pressure of 0.023 mm, using monochromatic radiation from the iron arc. At 2750 A a radiation flux of about 360 ergs sec.$^{-1}$ gave an ion current of $4\times10^{-10}$ amp. The mean value of $k$ at 2750 was $2.9\times10^{-10}$ and from figure 4 we compute $k=1.1\times10^{-19}$ at the limit 2968 A.

### 3. POTASSIUM

The efficiency of the space-charge method was low for potassium, but the radiation from iron and brass arcs gave an effect that was easily measured for wave lengths shorter than 2800 A. Figure 5

shows our relative values in comparison with results of Williamson [16] and of Lawrence [17] obtained by the vapor-jet method. In our experiments the potassium was at 200° C. corresponding to a vapor pressure of about 0.005 mm. The potassium limit is at 2856 A. The results are in striking contrast to the sensitivity curves of caesium and rubidium. The radiation at 2750 A is very intense, so that the low sensitivity near the limit could be measured with fair precision. Absolute measurements were not attempted.

### 4. ABSORPTION MEASUREMENTS WITH CAESIUM VAPOR

A direct measurement of the absorption coefficient of caesium vapor for the 3130 line of mercury was attempted, though the apparatus was not well adapted to the experiment. A quartz absorption cell 30 cm long was kept at about 230° C., while an appendix containing caesium was successively heated and cooled. The space-charge tube was used to measure the intensity. The best series of readings with caesium heated to 218° C. and cooled to 100° C. gave a change of 2.4 per cent. This corresponds to a value of $k=3\times10^{-19}$ at 3130 A. Other readings ranged from 1.8 to $6\times10^{-19}$, but accidental fluctuations were always of the same magnitude as the absorption effect. The photoionization method gave $k=1.85\times10^{-19}$ at 3130 A.

## IV. CONCLUSIONS

As concerns the shape of the photosensitivity curves there are only minor discrepancies between results obtained by different methods and observers. For caesium and rubidium the curves drop sharply from the limit to a value of 15 and 5 per cent, respectively, at 600 A beyond the limit; for potassium the sensitivity is low at the limit and rises toward the violet. It has been suggested that the former type of curve is characteristic of atomic vapor and the latter of molecular vapor.[18] If so, the gradual rise at the violet end of the caesium curve and practically the entire effect in potassium may be of molecular origin. It is interesting that our measurements in stagnant potassium vapor at 0.006 mm give nearly the same results as were obtained by Lawrence [19] and Williamson [20] in jets issuing from a furnace at 400° C. (partial pressure 4.5 mm), for it has been suggested that this type of curve was characteristic of the jet method. These results are not inconsistent with the form of the absorption curves found by Ditchburn.[21]

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[16] See footnote 5, p. 304.
[17] See footnote 4, p. 304.
[18] See footnote 4, p. 304.
[19] See footnote 4, p. 304.
[20] See footnote 5, p. 304.
[21] See footnote 2, p. 303.

The upward slope on the short wave length side of the caesium sensitivity curve may depend on impurities, for different observers are not in agreement on this, but the abnormal sensitivity at 2537 A found by Lawrence and Edlefson [22] and ascribed to mercury vapor, was absent in this work. The fact that the form of our curves was quite independent of pressure is hard to understand if either caesium molecules or impurities influence the observations.

There is a surprising discrepancy between the absolute values of $k$ for caesium. Our experimental magnitudes are comparable with those of Little,[23] except as concerns the observed ion current, and for that his values are of the order of one-fiftieth of ours for the same temperature. A low value can be obtained if insufficient caesium is in the tube or if there is a slight air leak. Little's measurements were in many respects more careful than ours, but various control experiments should rule out the possibility of a mistake in order of magnitude in our result.

There is a considerable variation in vapor-tension data of different observers sufficient to change the value of $k$ for caesium by a factor of two. Experimental values [24] are given both for low pressures and high pressures with no measurements in the intermediate ranges we have used. Plots of log $p$ versus $1/T$ give nearly parallel lines for different sets of observations, so that the best line through all sets has a slope quite different from all observed slopes. We found that values of $k$ computed from this mean line showed a systematic variation with pressure which was removed when we used values of pressure based on any single set of observational data. Because of the uncertainty in pressure the difference in the values of $k$ for caesium and rubidium has little significance.

The direct-absorption measurement in caesium is of interest in spite of its inaccuracy, for it indicates that the efficiency of photo-ionization is at least of the order of unity The existence of molecules with absorption limits on the short wave length side of the atomic limit would cause absorption without ionization superposed on the photo-electric absorption near the limit. In view of the possibility that there are such molecules it is important to know whether there are enough of them to seriously alter the atomic effect, for then the value of $k$ has little significance. We conclude that this is not the case in caesium vapor at 0.1 mm.

The paper by the authors on recombination spectra of ions and electrons [25] includes data for the estimation of the effective cross section $q$ of a caesium ion for recombination into the $2P$ level. These

[22] See footnote 8, p. 304.
[23] See footnote 7, p. 304.
[24] See footnote 13, p. 307.
[25] See footnote 9, p. 304.

give a value $q=6.2\times10^{-21}$ for electrons with a velocity corresponding to 0.19 volt. (There is a numerical error in the published value.) By the principle of detailed balance

$$\frac{k(\nu)}{q}=\frac{1}{a}\frac{m^2v^2c^2}{2h^2\nu^2}$$

where $m$ and $v$ are electron mass and speed and $\nu$ is the frequency at an interval beyond the limit corresponding to $v$. The a-priori probability, $a$, is taken as 3 for the superposed effect of the two $P$ levels. This gives $k=3\times10^{-17}$ near the 2 $P$ limit for excited atoms in the 2 $P$ state. Beyond the 1 $S$ limit the intensity of the recombination light is of the order of 1 per cent of that beyond 2 $P$. This makes $q=1.5\times10^{-23}$ for 0.19 volt electrons recombining into 1 $S$ and $k=0.5\times10^{-19}$. The result involves values for the ion and electron concentrations which are quite uncertain, so that an agreement in order of magnitude with the observed value of $2.3\times10^{-19}$ is all that can be expected. We have noted in the introduction that the computed variation of $k$ with wave length for atoms in the 2 $P$ state was very different from that observed for atoms in the normal 1 $S$ state. It is desirable that the recombination measurements be extended and improved in absolute value so as to test more rigorously the applicability of the principle of detailed balance to this class of phenomena.

It is of interest to compare the absorption in the continuous region with data on line absorption. The significant datum in the case of an absorption line is the integral of the absorption across the width of the line, and in practice it is usually necessary to broaden the line by pressure or some other means to make the width measurable and to reduce the absorption at the center. Sugiura [26] has shown that the integrated absorption over a frequency interval including one or more lines will approach in value the integral over an equal interval beyond the limit when the intervals are taken close to the limit. Waible [27] has recently published measurements for the higher series lines in caesium with lines broadened by high vapor pressure (10 to 30 mm.) and gives values of $\int k d\nu$ for lines from the fifth to the fourteenth, inclusive. If intervals $\Delta\nu$ are chosen equal to the successive intervals between lines so as to include one line in each interval, one finds that

$$\frac{\int k d\nu}{\Delta\nu}=0.48\pm0.01\times10^{-19}$$

for all lines from the eighth to the fourteenth. This constancy is a consequence of the approximations that both the integral and $\Delta\nu$ are inversely proportional to the cube of the total quantum number

[26] Sugiura, J. de Phys., **8**, p. 113; 1927.
[27] Waible, Zeits. f. Phys., **53**, p. 459; 1929.

when the number is large. This is not in satisfactory agreement with our value $k=2.3\times10^{-19}$ at the limit. Other measurements[28] on the first four lines of caesium where lines were broadened by high pressures of an inert gas give values about ten times greater than Waible's. From the same source $k$ for other alkalies can be computed with the following results: Na (Trumpy) $8\times10^{-19}$, Rb $5\times10^{-19}$.

Various theoretical derivations of the absorption beyond series limits of the hydrogen atom give values which are about a hundred times greater than our value for 1 $S$ of caesium. Thus, Oppenheimer[29] gives for hydrogen in the $2_2$ state

$$k=0.99\times10^{-12}\frac{\lambda^{3.4}}{\lambda_0^{2.4}} \qquad k=3.6\times10^{-17} \text{ at } \lambda_0$$

This is quite similar to the result computed from recombination data for caesium atoms in the 2 $P$ state ($k=3\times10^{-17}$ at the limit and decreases as $\lambda^4$).

Sugiura[30] gives approximate theoretical computations for sodium line absorption which are in satisfactory agreement with experiment. The theory assumes that all one-electron systems have the same total absorption integrated over all lines and the continuous spectrum, and it is shown that the low absorption beyond the limit is associated with an abnormally high absorption for the first line in the principal series as compared with the hydrogen value.

We are greatly indebted to Doctor Coblentz and to Mr. Stair for generously advising and assisting us in making the radiation measurements and for supplying apparatus and calibrated standards for our work.

## V. ADDENDUM

Since this work was completed, Lawrence and Edlefson have reported before the Physical Society[31] relative measurements of photoionization in caesium, rubidium, and potassium made by the space-charge method. The results for rubidium are very similar to ours; their caesium curve is more nearly like rubidium. For potassium their measurements show a well-defined peak at the principal series limit in addition to the increase further in the ultra-violet with evidence of a maximum near 2300 A, which is roughly three times the maximum at the series limit.

WASHINGTON, May 8, 1929.

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[28] Wolf and Herzfeld, Handbuch der Phys., **20**, p. 616.
[29] Oppenheimer, Zeits. f. Phys., **41**, p. 268; 1927.
[30] Sugiura, Phil. Mag., **4**, p. 495; 1927.
[31] Proc. Washington Meeting, Apr. 18–20; 1929.

# MAKING THE GLASS DISK FOR A 70-INCH TELESCOPE REFLECTOR

By A. N. Finn

ABSTRACT

Details are given of the procedure followed in making a glass disk 70 inches in diameter and 11 inches thick for use as a reflector in an astronomical telescope. These include the making of the pots, the modifications in the construction of the melting furnace, the construction of the mold and annealing furnace, the methods of measuring and controlling temperatures, the melting and casting of the glass, essential data on annealing the glass, the determination of the quality of annealing (the amount and distribution of the residual strain), and the drilling of the hole to accommodate a Cassegrainian mounting.

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## I. INTRODUCTION

The art of making glass probably dates from about 1600 B. C., when the Egyptians made crude glass vessels by gathering molten glass on clay or sand forms. The glass of that period was similar to the older enamels or glazes and was generally more or less opaque. It contained many inclusions of undissolved matter and was generally highly colored either by iron compounds unavoidably present in the raw materials from which the glass was made or by some

coloring matter, such as minerals containing cobalt and copper intentionally added.

During succeeding centuries, as the art was developed, glass making became an established industry and the quantity and quality of the product improved. It has only been within the last century, however, that glass making became a problem of scientific interest, and the resulting improvements made soon led to the production of glasses free from color, opaque inclusions, and other visible defects, and to the manufacture of glasses having certain definite compositions. Such glasses have definite optical properties, and some of them are used in the construction of precision optical instruments and consequently are referred to as optical glasses.

Marked progress in the optical-glass industry dates from about 1850. Since then the developments have been such that optical instruments containing small pieces of glass are now quite common. Optical instruments, such as large telescopes, are, however, not common, probably because of a lack of demand for them, but unquestionably this in turn is to a great extent the result of the cost of the equipment and difficulties met in securing suitable glass. The number of large telescopes in the world is relatively small, and among these the 40-inch refractor at the Yerkes Observatory and the 100-inch reflector at Mount Wilson are the largest of their respective types.

The manufacture of glass for ordinary optical instruments is a well-established but limited industry, there being probably not more than 10 optical-glass plants in the world. This industry is, however, very necessary for the maintenance and development of our commercial and scientific enterprises, since a lack of suitable glass for making microscopes, cameras, surveying instruments, range finders, etc., would greatly hamper all work in which such instruments are used.

Difficulties which are of minor importance in making glass for smaller optical instruments become more serious as the size of the instrument, and consequently the size of the glass, is increased. The most serious difficulties met in producing large pieces of glass include properly melting the amount of glass required, transferring it to a mold to produce the desired shape while maintaining the necessary quality, and finally cooling it at rates such that it does not crack and that it will be well annealed (free from disturbing internal stresses).

The proper cooling (annealing) of large pieces of glass is probably the most exacting and tedious phase of the work, and this can not be successfully accomplished without knowing the thermal properties of the glass and adequately controlling the temperatures of the annealing furnace.

## II. PLAN OF THE WORK

Because of the lack of definite information on methods of making satisfactory pieces of glass which could be used for large telescope reflectors, the Bureau of Standards became interested in the problem and undertook to make the glass for a 61-inch reflector. Such a reflector should be 10 inches thick when finished and, if made of a borosilicate crown glass, would weigh approximately 2,960 pounds.

In planning this work it was decided to make the disk about 70 inches in diameter and 15 inches thick in order to make ample allowance for faulty edges. A piece of glass having these dimensions would weigh approximately 5,000 pounds. The first plan decided on involved the melting of the glass in one large pot and cooling it in such a way that the glass would not crack and so that the pot would spall from the glass. Having obtained a piece of glass of sufficient size, it would then be put in a mold of the required diameter and an electric furnace built around it so that the glass might be slowly heated until it became soft enough to flow and fill the mold uniformly; after this it would be cooled to the proper annealing temperature and annealed. Four attempts to obtain the initial piece of glass were unsuccessful because the glass cracked each time during cooling. This method was then discarded and a different one tried. In the new method the glass was to be melted in a pot provided with a hole near the bottom, this hole being stopped with a water-cooled metal plug. When the glass was properly melted a metal trough was to be placed between the pot and mold, the plug removed, and the glass allowed to flow into the mold where it would be ready for cooling and annealing.

The successful application of this method depended on three major conditions: (1) Preventing the glass from leaking through the hole in the pot by means of a water-cooled metal plug, (2) adequately controlling the flow of glass from the pot to the mold, and (3) obtaining a satisfactory friable lining for the mold. The advantage of this over the first method tried was that only one cooling of a large and relatively thin slab would now be required, whereas the first one offered three additional hazards: (1) The possibility of the glass cracking during the initial cooling on account of its great thickness, (2) the possible failure of the pot to break away from the glass, and (3) the possibility of cracking the glass during reheating for softening in the mold.

## III. THE POTS

Before any of this work was done it was necessary, of course, to make pots sufficiently large to hold the amount of glass required. The dimensions of the furnace available for the purpose determined, to a large extent, the actual dimensions of the pots, which had, after

burning, a depth of 33.3 inches and an internal diameter of 47 inches at the top and 43 inches at the bottom. The pot walls were 3½ and 4½ inches thick at the top and bottom, respectively. (See fig. 2 for a cross-section view of the pot.) The pots were "cast" in a plaster of Paris mold by a method developed at this bureau [1] and used in making all of our glass pots, but, since these large pots were more than six times the size of our ordinary melting pots, it was necessary to make a larger mold for casting them. The sides of the mold consisted of three vertical sections, the core of a monolithic hollow piece, and the base of a single solid slab. About 10 tons of plaster of Paris were used in making the forms, the model, and the finished mold. Each pot was made from 2,500 pounds of domestic clays and feldspar blended in the following proportions:

| | Per cent |
|---|---|
| Feldspar | 8 |
| Tennessee ball clay | 30 |
| Kentucky ball clay | 21 |
| North Carolina kaolin | 23 |
| Delaware kaolin | 9 |
| Georgia kaolin | 9 |

The clays were thoroughly blunged with water to which 0.6 per cent of water glass was added, and the resulting slip was poured into the mold. After standing in the mold for two days the core was withdrawn, and the next day the sides of the mold were removed. The pot was then slowly dried until most of the water had evaporated; finally it was air-dried for six months before being used.[2]

## IV. THE MELTING FURNACE

The furnace (fig. 1) was an ordinary, down-draft, gas-fired, single-pot furnace of the regenerative type modified by building a series of flues in the floor, each alternate flue being connected at one end with the space over the checkerwork, the other end opening into the combustion chamber. By means of temporary dampers (*S*, fig. 2) over the checkerwork, the heated air and products of combustion were forced to travel down the outside of the pot and through the flues underneath it, thus establishing a more uniform temperature around and under the pot during the initial burning than would have been obtained by radiation alone; after the pot had been burned to about 1,000° C. the temporary dampers were removed, and the furnace was then operated as an ordinary regenerative furnace.

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[1] The Equipment of a Casting Plant for the Manufacture of Glass Pots, by F. H. Riddle, J. Am. Ceram. Soc., 2, p. 8; August, 1919.

[2] A more complete description of the making of these pots can be found in Ceramic Ind., pp. 186-187; September, 1924.

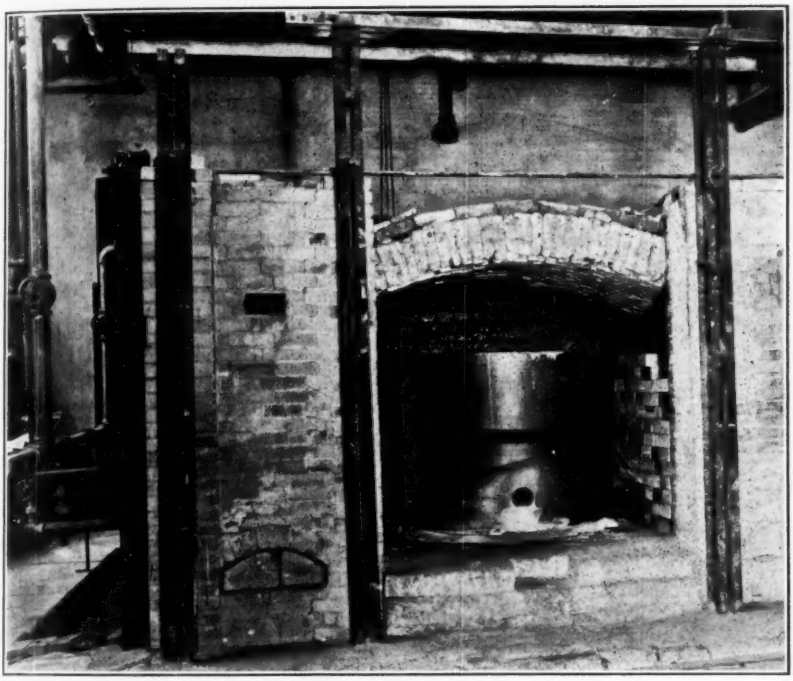

FIGURE 1.—*The melting furnace (front removed) and pot after the glass had been made*

The shiny surface on the bricks which appears just above the pot is "glass" impregnated with corundum crystals. This is a characteristic condition in glass-making furnaces, and is the result of fluxing action produced by volatilized compounds, such as soda, from the glass on the refractories.



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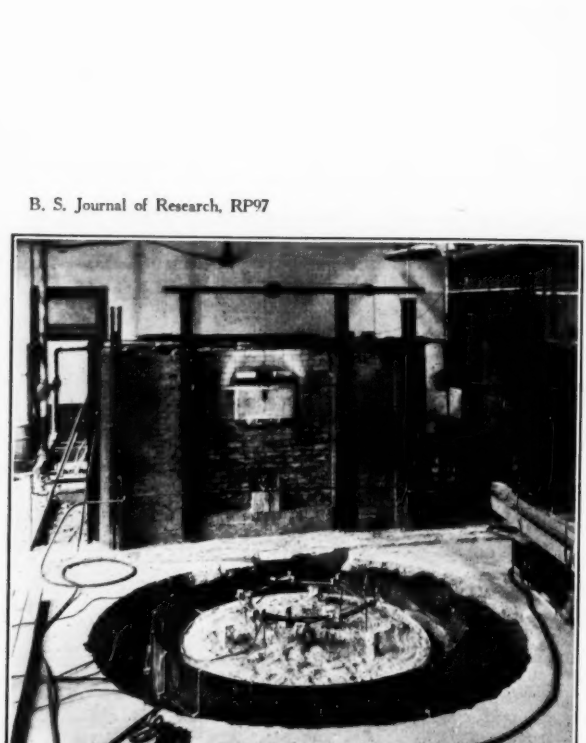

FIGURE 3.—*The melting furnace (back), the assembled mold and annealing furnace (center) and the trough (right)*

This picture was taken just before the lid was removed preparatory to casting the glass.

## V. ESSENTIALS OF METHOD FINALLY USED

In the original method the pots were used as cast, but in the method finally employed it was necessary to provide a hole near the base of the pot which could be plugged during the melting of the glass and through which the glass could be tapped when the plug was removed. This hole was conical, the small diameter being on the inside of the pot, and it was made before the burning of the pot was started. The plug for this hole consisted of a hollow, truncated cone turned from a block of steel and provided with two pipes in the larger base to permit rapid and continuous circulation of water in the plug. The dimensions of such a plug must, of necessity, be based on the dimensions of the hole after the pot is burned, but since the shrinkage of the pot

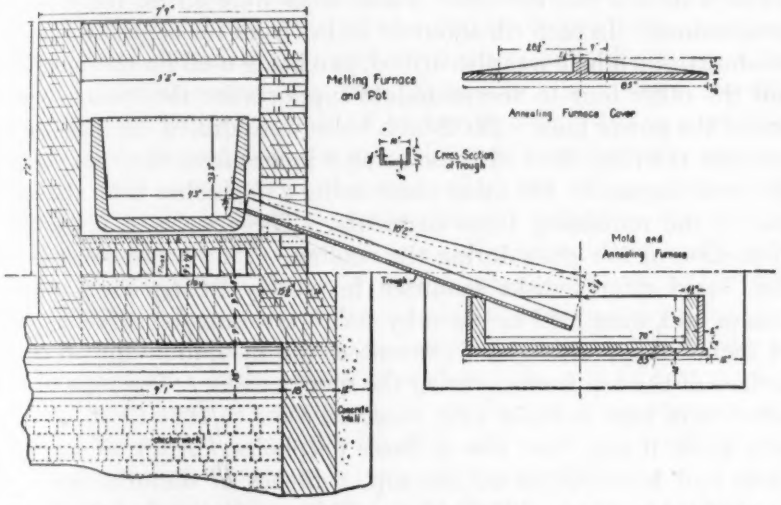


FIGURE 2.—*Essential features of equipment used in making a 70-inch disk of glass*

body from air-dried to burned condition was known to be about 8.5 per cent, the dimensions of the plug required could be predicted fairly accurately from those of the hole before burning. Furthermore, it is not essential that the dimensions of the hole and plug should be the same, but the tapers of both should be practically the same, and the small diameter of the plug should be such that the plug, when firmly seated in the hole, could not reach beyond the inner surface of the pot. In this particular case the hole in the pot was made about 5 inches in diameter at the large end and had a taper of about 8½°. The plug was about 3¾ inches long.

The combined mold and annealing furnace (fig. 2) into which the glass was tapped consisted of four essential parts: (1) The metal forms, (2) the lining, (3) the heating elements, and (4) the insulation.

The metal forms were made of cast iron and consisted of a top, a ring, and a base. The top, which was 85 inches in diameter and 2 inches thick, was flat on the lower surface and had six radial ribs on the upper surface; these ribs were joined at the center of the upper surface in a cylinder 5 inches high and 5 inches in diameter and tapered to the surface of the top at its edge. (Fig. 2.) The ring was 79 inches in diameter (inside), 2 inches thick, and 19 inches high. The base was the same as the top except that it was not ribbed. The base, ring, and top were drilled and tapped for eyebolts to facilitate handling.

Six equidistant holes, one-fourth inch in diameter and approximately 2 inches deep, were drilled in the edges of both the base plate and of the top. Six similar holes were drilled into the ring halfway between its top and bottom. These holes were all for thermocouple installations. In each rib about 20 inches from the center of the top a one-fourth-inch hole was also drilled, two being used for thermocouples and the other four to accommodate supports for the frame carrying one of the power lines. Six 2-inch holes were drilled through the top between the ribs, three of these being 9 inches from the center and in alternate segments, the other three being 28½ inches from the center and in the remaining three segments. These were to permit local inspection of the glass during the cooling. They were covered with thin fused quartz disks mounted in closely fitting sheet asbestos frames and were held in place by hollow rectangular fire-clay tubes 14 inches long, which also prevented the insulation spread on the surface of the top from covering the quartz disks. The tops of these tubes were kept covered with sheet asbestos so that there would be very little, if any, heat loss at these peepholes. After all the equipment had been placed on the top, 5 inches of diatomaceous silica were added for thermal insulation. This was retained at the edges by strips of asbestos board fastened around the casting with a steel strap.

The lining for the mold was very fine grained, friable brick (2¼ by 4½ by 9 inches) made from ground, bonded, and burned diatomaceous silica. The heating elements consisted of spirals of No. 16 gage nickel-chromium wire mounted in grooved refractory plates (9 by 14 inches), each element having a resistance of about 13.5 ohms.

## VI. PLACING AND ASSEMBLING THE MOLD

Since the floor of the melting furnace was only slightly higher than the floor of the furnace room, a pit to contain the mold was dug in front of the furnace so that the glass might flow freely from the melting pot to the mold. This pit was about 15 feet in diameter and 4 feet deep.

A sheet-metal "can" without top or bottom, 109 inches in diameter and 48 inches deep, was placed in the pit, and inside the can a layer

of carefully packed diatomaceous silica 1 foot thick was placed on the ground. Eighteen heating elements were so placed on the insulation that they would be symmetrically arranged under the base plate and be so distributed that each element would heat approximately an equal area of the base plate. The leads from each element were ultimately connected to the power line after bringing them through holes previously drilled in the can at the proper level. The base plate was put in position over the heating elements and the ring put in place concentrically with respect to the plate. Eighteen heating elements were fastened around the outside of the ring, and the leads from each one of these elements were also carried out through small holes in the can. The space between the ring and the can was then filled with powdered insulation which was not packed.

The bricks for the bottom of the mold were laid without mortar and in the herringbone style on the 4½ by 9 inch face, care being taken to have the bricks very well fitted and to keep the upper surface level. After the entire base was covered in this manner, the sides were laid up to give a wall 15 inches high and 4½ inches thick. Each brick in the wall was so cut at the ends that the wall conformed nearly to the curvature of the ring. The top of the furnace was provided with 18 heating elements placed similarly to those under the bottom, the leads from each plate being brought up to a wooden framework supported about 18 inches above the casting by rods fitted into the holes drilled in the ribs of the casting. (Fig. 3.)

The channel which was used for conveying the glass from the pot to the mold was 5 inches deep, 10 inches wide, and 10 feet 2 inches long. It was made by cutting off both flanges from one side of a standard 10-inch H beam (55 lbs./ft.). It was very carefully sand blasted to remove all traces of rust and scale so that the glass might flow without being contaminated with iron oxide and without the possibility of scale snapping off as the metal became heated. The channel was supported near both ends from an overhead crane with flexible steel cable over pulleys in such a way that the elevation of either end could be controlled independently.

## VII. TEMPERATURE MEASUREMENTS

The temperature of the melting furnace was determined both with thermocouples connected to a portable potentiometer and with an optical pyrometer. During the initial burning of the pot 10 chromel-alumel thermocouples were used—4 near the upper edge of the pot, 4 near the bottom (outside), 1 inside the pot near the bottom, and 1 in the sand under the pot. When the temperature of the furnace reached 1,000° C., these couples were removed and a platinum to platinum-rhodium couple was used during the final burning of the pot. This couple projected through the back wall of the furnace and was

about 18 inches above the furnace floor. It was also used during the melting of the glass to indicate constancy of temperature, while the optical pyrometer was relied on to indicate the average temperature.

Temperatures of the annealing furnace were measured with chromel-alumel thermocouples connected to a portable potentiometer. The installation of the thermocouples in each section of the furnace was made before the insulation above the base plate was put in place. In order to prevent breakage of the thermocouple protection tubes by a possible settling of the furnace, they were not brought out through the sides of the can but the couples and tubes were made long enough to reach from their positions in the castings to the upper edge of the can. The thermocouples and protection tubes in the ribs and sides of the top extended straight up through the insulation. One thermocouple whose hot junction was about one-fourth inch below the upper surface of a brick in the base of the mold and about 15 inches from the ring was brought out between the ring and the base and carried up to the edge of the can. In this case two tubes were used, one for each wire, because it has been found that protection tubes not closed at the inner end and extending through insulating material are liable to collect a small amount of water in the cold end and produce erratic readings; but if two tubes are used, there is no reason to believe that collected water in either or both affects the readings. All couples were connected with a switchboard by means of compensating or extension lead wires, and proper cold junction corrections were made. The cold junction for this work was in a pipe driven about 12 feet into the ground where the temperature did not change more than 2° C. during the entire operation.

Although 22 couples were installed, it soon became evident that it was not necessary to use more than 5 of them, because the temperature of the furnace was sufficiently well indicated by these 5. However, occasional checks were made on all couples, using an independent short connection to the potentiometer.

## VIII. HEATING CURRENT INSTALLATION

Arrangements were made to supply, if necessary, a maximum of about 50 kilowatts to the annealing furnace in case the preheating of the furnace and the heat from the molten glass did not bring the furnace temperature well above 600° C. From the main switchboard three lines were run to the furnace. Two of them were placed around the can, one at about the elevation of the base plate and the other opposite the middle of the ring. The third extended to the framework above the top of the furnace. Since the leads from each heating element were brought outside of the furnace, it was possible to connect each element across the leads or to connect them in series as current requirements dictated. By making series connections,

the 18 heating elements on each of the three circuits could be connected in groups of 2, 3, 6, 9, or 18 elements each and thus reduce the use of external resistance without disturbing temperature distribution. Power consumption of the furnace was measured with ammeters and voltmeters so that the amount of energy being used in each of the three sections of the furnace could be determined and, by means of external resistance, controlled. A diagrammatic plan of the electrical installation is shown in Figure 4.

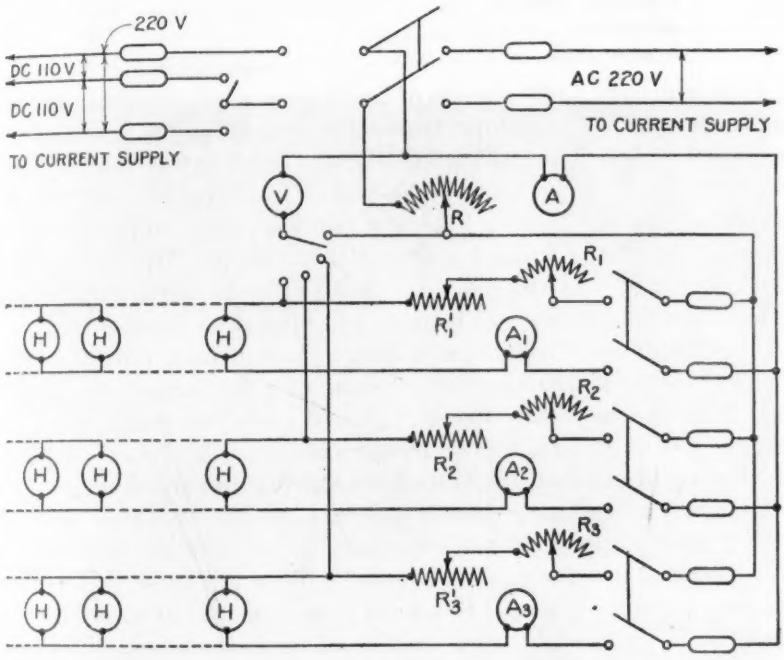


FIGURE 4.—*Diagrammatic plan of electrical installation*

*H*, Heating elements; *V*, voltmeter; *A*, ammeter; *R*, variable external resistance.

## IX. PROCEDURE

All the equipment being ready, the burning of the pot was started. Since it was necessary to burn the pot without cracking it and since there was no method of adequately examining it after burning, it was necessarily burned very slowly, 30 days being required to reach a temperature of 1,425° C., at which the glass would be melted. After the pot was properly burned at this temperature the lower door in the furnace was opened, the water-cooled plug was fastened in place, and 1,000 pounds of cullet were put into the pot. This was sufficient to fill the pot to a point above the hole and plug, and, since no leak

was evident, the filling of the batch at the rate of 170 pounds per hour was started. The batch had the following composition:

| | Per cent |
|---|---|
| Sand | 49. 9 |
| Boric acid | 4. 3 |
| Borax | 17. 5 |
| Soda ash | 5. 7 |
| Potash (83 per cent) | 12. 0 |
| Barium carbonate | 1. 9 |
| Potassium nitrate | 6. 5 |
| Zinc oxide | 1. 9 |
| Arsenic oxide | . 3 |

Approximately 5,000 pounds of batch were used, requiring 31 hours to fill the pot. The temperature of the melting furnace was then reduced to 1,400° C., and the stirring of the glass was started. This was done by hand, using a water-cooled iron stirring rod, and was continued for six hours. The stirring was then stopped and the temperature of the furnace reduced to 1,350° C. While the batch was being filled and the glass was being stirred, current was applied to the annealing furnace to heat and to hold it at about 400° C.

Figure 5 gives some idea as to the equipment used during stirring. The water-cooled stirring rod projecting into the furnace can be seen, and also the temporary flooring, placed over the mold, which was necessary to provide access to the furnace from the front. The time which elapsed between the end of the stirring and the actual tapping of the glass was greater than it should have been because, after the stirring was stopped, it was necessary to remove the flooring, the rails on which it was supported, and the lid of the mold, and to swing the channel into place. If this work were repeated, it would be more desirable to tap the pot from the side of the furnace opposite to that used for filling and stirring, so that the stirring might be continued until the glass was being tapped. By so doing, the prospects of getting glass relatively free from cords would be improved.

## X. CASTING AND ANNEALING THE GLASS

When all arrangements were completed for tapping the pot, the supports for the water-cooled plug were released and the plug removed. The glass in contact with this plug was practically solid, and the molten glass did not begin to flow from the pot until the plug of cold glass became soft enough to yield. After it had become soft enough, the glass flowed very rapidly. As its level in the mold rose, the ends of the channel were adjusted so that one end remained practically fixed at the hole in the pot and the other was not permitted to touch the glass in the mold. (Fig. 6.) When practically all the glass

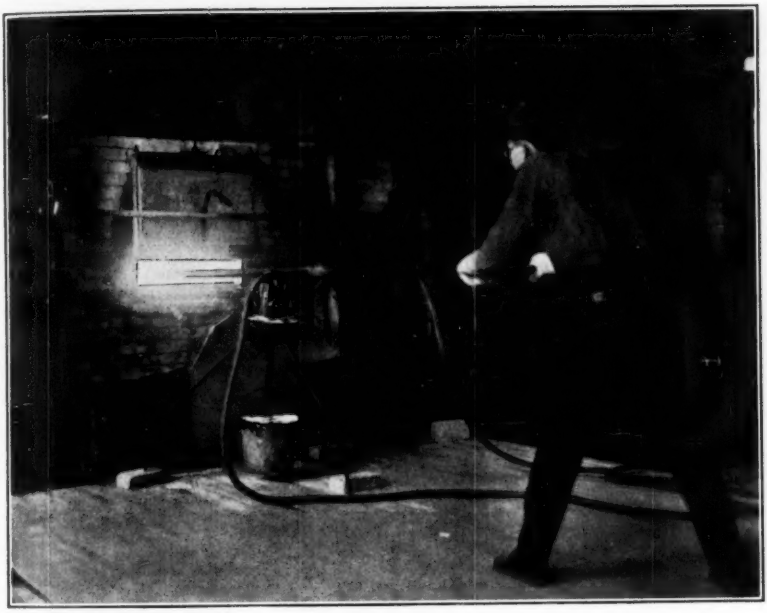

FIGURE 5.—*Stirring the molten glass to increase its homogeneity*

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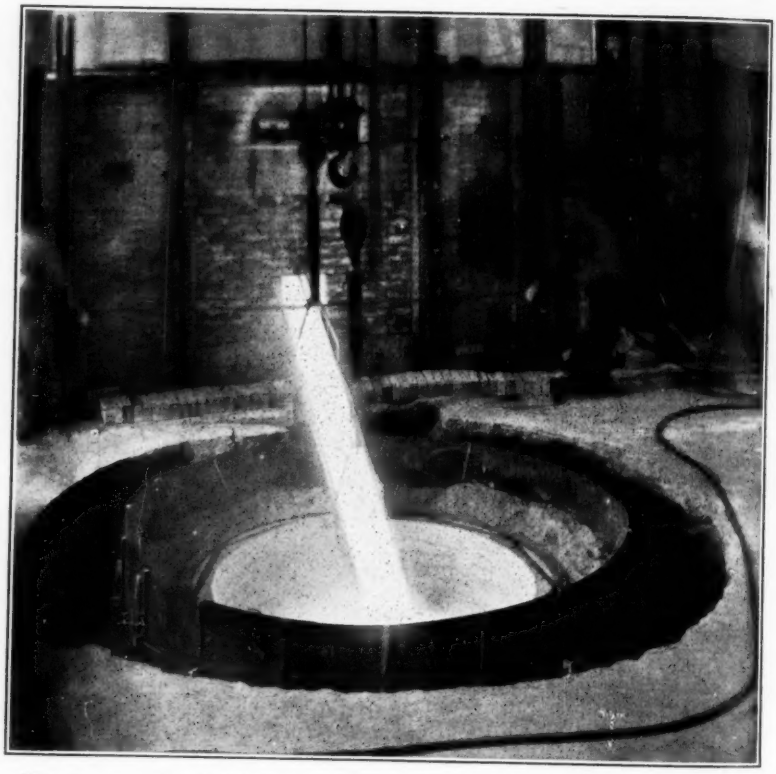

FIGURE 6.—*The initial flow of the molten glass from the pot to the mold*

was in the mold, the trough was removed and the lid was put in place as quickly as possible to conserve a maximum amount of heat furnished by the glass. This was not a good procedure, because shortly after placing the lid on the mold it cracked in two places on account of too rapid heating, but fortunately this produced no undesirable results.

The maximum temperatures ever indicated by the thermocouples were about 850° C. in a rib of the lid and about 1,000° C. immediately below the surface of the brick in the bottom of the mold. At this time there were only the original 5 inches of insulating material on the top, but 24 hours later about 3 inches more of insulation were added. Nine hours later additional insulation was added until all metal parts of the mold were covered to a depth of 12 inches. The heat loss was then so slow and the amount of heat to be lost by the glass before reaching 600° C. was still so large that it was not necessary to use heating current for about four days, at which time it was applied intermittently to equalize the temperature of the system.

Since borosilicate crown glass of the composition used in this work can be cooled safely to 600° C. without danger of breaking, it was cooled to this temperature as rapidly as the furnace construction would permit, but without allowing any part of the furnace to cool below 600° C. Eight days after casting the glass the average temperature of the furnace was 596.7° C.[3] Three days later the average temperature was 594.8° C. In the meantime the power supplied to the heating elements was 2,752 watts. During the next two months the heating current was gradually reduced until the temperature for annealing (461° C.) was reached.

This temperature was based on the data of Adams and Williamson,[4] who have shown that glass can be most efficiently annealed if the annealing temperature and subsequent cooling rates are properly chosen. Although the time required for stresses or strain in glass to disappear at temperatures just below the initial softening point of the glass is very short (say, two minutes), it increases rapidly as the annealing temperature is lowered. However, the authors cited demonstrate that glass can be cooled more rapidly after annealing at lower temperatures than it can be after annealing at higher temperatures. Ultimately, then, the time required for the whole operation (annealing and cooling) may be shortened if lower annealing temperatures and faster cooling rates are used. The most desirable

[3] Temperature measurements are probably correct within 2° C. Changes in temperature could, of course, be determined more accurately than this. The potentiometer scale was graduated in 0.1 mv, corresponding to 2.3° C., and could be read to 0.02 mv.

[4] J. Frank. Inst., **190**, pp. 597–631 and 835–870; 1920.

annealing temperature for a piece of glass of any thickness can be computed from the following equations:

$$\log A = M_1\theta_0 - M_2,\ A = \frac{0.047}{C},\ \text{and}\ C = 4.6\ Ba^2$$

in which $\theta_0$ is the desired annealing temperature (° C.), $a$ is half the thickness of the glass (in centimeters), and $M_1$, $M_2$, and $B$ are constants whose values for borosilicate crown glass are 0.03, 18.68, and $2.85(10)^{-7}$, respectively.[5] The time ($t$), in minutes, required for annealing or release of stress is found from

$$t = \frac{0.37}{A}$$

The initial cooling rate ($h_o$) after annealing is $\frac{5.2}{C}$ ° C. per minute, and the permissible cooling rate ($h$) at any temperature ($\theta$) below the annealing temperature ($\theta_o$) is obtained from

$$h = \frac{h_o}{2}(1 + 2^x),\ \text{in which}\ x = \frac{\theta_o - \theta}{20}$$

Since Adams and Williamson based their findings on pieces of glass much smaller than the piece being annealed, and since the validity of extrapolations from data obtained on small pieces to those for a much larger piece might be questionable, it was decided to anneal the glass about twice as long as computations indicated would be necessary and to cool it initially at about one-half the computed rate. The annealing temperature and cooling rates were based on a thickness of 31 cm, which was slightly greater than the actual thickness of the disk. It was felt that these safety factors would also offset the effect of any differences between the temperature of the glass and the metal castings in which the thermocouples were placed and of any inequalities in the temperature of the castings not indicated by the thermocouples.

The glass was annealed for 41 days at 461° C., the total power consumption during this time being 1,890 watts, distributed as follows: 660 watts on the bottom of the furnace, 444 on the ring, and 786 on the top. During the annealing period constant personal attention was given to the annealing furnace in order to adjust the heating current as frequently as might be made necessary by changes in the voltage of the power line. The care with which this was done is shown by the following: The average temperature of the furnace during the 41 days used in the annealing operation was 461.5° C., the

[5] In this presentation, $A$ may be called the "annealing constant" and depends on the rate at which glass anneals when held at constant temperatures. $M_1$ and $M_2$ are constants obtained by plotting $\log A$ (10)[4] against temperature. $B$ is a function of the birefringence produced by a given stress. In a paper of this kind a more comprehensive mathematical discussion would be out of place, and the reader is referred to the work previously cited.

maximum average[6] on any day was 463.4° C., and the minimum was 458.3° C.

## XI. FINAL COOLING

Assuming that the glass at the end of the annealing period was free from strain, it still was necessary to cool it so slowly that no great amount of strain would be present at the end of the operation. Although the data of Adams and Williamson indicated that an initial cooling rate of 2.33° C. per day and a maximum rate of 60° C. per day would be permissible, it was decided that the initial cooling rate used should be about 1° per day and the maximum rate should not exceed 10° per day; and, if possible, this maximum would be restricted to 6° per day.

Cooling was first accomplished by reducing the power consumption 5 per cent of that required to maintain the constant annealing temperature and by applying this reduced amount of power until the temperature drop became definitely slower than it was at the beginning. Generally five days were required to show a definite decrease in the cooling rate. Sixteen similar cuts of 5 per cent each were made in the heating power, after which the amount of each cut was reduced to 2.5 per cent until the power used was only 5 per cent of that used for annealing. Finally, cuts were made at the rate of 1 per cent. It was thought that the very slow cooling at lower temperatures was necessary because, in the final equalization of any unknown temperature differences in the system, temporary strains might develop in the glass which would be sufficient to break it, and this, of course, must be avoided if possible

The uniformity of temperature maintained during cooling is indicated by the following table:

| | | Temperatures at different parts of the furnace | | | | | Average temperature | General average temperature |
|---|---|---|---|---|---|---|---|---|
| | | ° C. | ° C. | ° C. | ° C. | ° C. | ° C. | ° C. |
| May 24 | Top | 610.5 | 611.7 | 611.7 | 612.8 | ---------- | 611.7 | 611.7 |
| | Ring | 614.0 | 614.0 | 611.7 | 612.8 | ---------- | 613.1 | |
| | Base | 610.5 | 610.5 | 608.1 | 608.1 | 614.0 | 610.2 | |
| July 8 | Top | 465.7 | 466.4 | 466.4 | ---------- | ---------- | 466.2 | 466.7 |
| | Ring | 468.7 | 468.0 | 468.0 | 468.7 | ---------- | 468.4 | |
| | Base | 466.4 | 466.4 | 465.3 | 465.3 | 463.4 | 465.4 | |
| Nov. 19 | Top | 176.9 | 176.7 | 175.7 | 177.4 | 178.2 | 177.0 | 176.8 |
| | Ring | 177.2 | 176.9 | 176.9 | 177.2 | ---------- | 177.1 | |
| | Base | 176.2 | 176.7 | 176.0 | 176.5 | ---------- | 176.4 | |

Temperatures for record were usually read only once a day. The average temperature for the day was obtained by averaging all temperatures recorded on that day.

## XII. INSPECTION DURING COOLING

When an attempt was made to project light through one of the peepholes in the top of the furnace and view the glass through another, it was found to be practically impossible because the lower surface of the quartz disks had been slightly etched by volatile materials, such as alkali or boric oxides from the hot glass. After the disks were removed, resurfaced, and replaced, illumination and inspection of the glass through these peepholes was very easily accomplished, but the glass could only be examined in the relatively small parts immediately below the peepholes. However, a sufficiently good view could be obtained to justify the conclusion that the glass was still intact.

Four and one-half months after the cooling from the annealing temperature started, the furnace and its contents were practically at room temperature. The surface insulation and the lid of the furnace were then removed and the glass was found to be in satisfactory condition. The disk was 69.75 inches in diameter, 11 inches thick, and weighed approximately 4,000 pounds.

## XIII. QUALITY OF ANNEALING

The disk was removed from the mold and the bricks, which were superficially fused to the glass, were stripped off. (Fig. 7.) This left the sides and bottom of the disk opaque, because the glass had partially fused to the bricks. Before an examination for strain could be made it was necessary to grind the bottom of the disk with carborundum and water to remove the adherent particles of brick. The disk was then placed on edge in a wooden frame and the ground surface coated with a heavy mineral oil which rendered it sufficiently transparent to permit examination with polarized light. The magnitude of the residual strain was measured by examining small sections of the entire disk. (Fig. 8.) The distribution or symmetry of strain was determined by projecting a divergent beam of polarized light against one surface of the disk and examining it through a nicol prism from the opposite side. In this case the light source and observer were each about 30 feet from the disk, and the entire disk was visibly through the nicol prism. The interference figure was very symmetrical with respect to the disk, intersecting exactly in its center, and the maximum strain detected produced a relative retardation equivalent to about 6 $m\mu$ per centimeter of thickness. This is well within the maximum strain permissible in optical glass used for optical instruments of highest precision.

## XIV. DRILLING THE HOLE

Large reflecting telescopes are generally equipped with one of three types of mountings—namely, the prime focus, Newtonian, and Cassegrainian. The last of these requires a hole in the center of the

FIGURE 7.—*The disk as it appeared after removing most of the adhering bricks from the side*

It is still resting on the bottom bricks and cast-iron base.

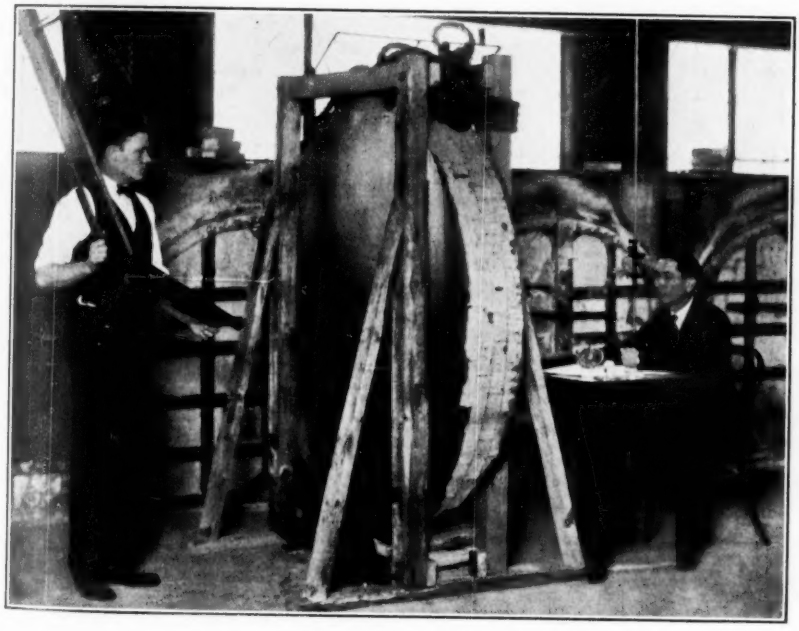

FIGURE 8.—*Examining the glass for strain*

FIGURE 9.—*Dr. George K. Burgess, Director of the Bureau of Standards (right), and Dr. C. C. Crump, Professor of Astronomy at Ohio Wesleyan University, examining the disk after the hole had been cut*

The "core" is resting on the surface of the disk.



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reflector so that light concentrated by the reflector may be returned by a secondary mirror to a point back of the reflector. This hole does not interfere with the use of the other mountings, but without it the Cassegrainian mounting could not be used. Hence, it was decided to drill an 8-inch hole in the center of this disk, although several authorities said this could not be done without a very great possibility of breaking the disk. A possible cause of breakage during this operation lies in the fact that in cutting a hole the surface of the cut is very finely fractured; the fractures, being near the center of the disk and in the zone of high tensional stresses, may easily become the starting point of serious cracks if the glass is not sufficiently free from strain.

The drilling was accomplished by attaching an 8-inch brass tube to a radial drill press and by slowly cutting out a core, using water and 150-mesh carborundum in the cut. About 70 working hours were required to cut this hole. (Fig. 9.) It might have been done in much less time, but, by working slowly, a partial adjustment of the glass to unbalanced stresses could take place and the possibilities of excessive local heating would be reduced.

This operation was conducted without any apprehension of possible cracking of the glass, although precautions were taken to protect the optician working on it in case the glass did "explode." These precautions consisted of bedding the glass on sand and surrounding it entirely with sand contained in the cast-iron ring used in the mold. The top was covered with 2 by 12 inch planks and ½-inch sheet steel, except for the hole through which the cutter passed.

## XV. CONCLUSION

Although the manufacture of large disks of glass to be used for reflectors in astronomical telescopes is ordinarily considered a difficult undertaking, it seems from the foregoing discussion that such is not the case, provided sufficient time is taken to anneal and cool the glass properly.

Based on the results obtained in the work just reported, it seems quite possible that American manufacturers can easily go into this field of endeavor, if occasion presents itself, with reasonable assurance that their efforts will be successful and that they need not spend much time and money, if any, in developing a satisfactory method of operation.

WASHINGTON, February 16, 1929.

# EFFECT OF WATER ON EXPANSIONS OF CERAMIC BODIES OF DIFFERENT COMPOSITION

By H. G. Schurecht and G. R. Pole[1]

**ABSTRACT**

The effects of the composition of some ceramic bodies on their moisture expansions were investigated. With the exception of mixtures containing blast-furnace slag or magnesite, most bodies with high absorptions developed considerable expansion. Bodies with low absorptions composed of pure clay and feldspar developed practically no moisture expansion, while those composed of clay, feldspar, and more than 20 per cent silica developed considerable moisture expansion. By adding magnesite the resistance of a ceramic body to moisture expansion was improved. Additions of whiting, iron oxide, and titanium oxide were not as beneficial as magnesite. It was found that crazing caused by moisture expansion of bodies can usually be prevented by proper manufacturing control.

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## I. INTRODUCTION

It has been shown [2] that a type of delayed crazing found on glazed ceramic ware may, in certain cases, be caused by a gradual expansion of the body during storage or use produced by the action of moisture while the glaze generally does not expand in the same manner. This type of expansion may be called "moisture expansion" to distinguish it from other types. It has, furthermore, been shown in the publication referred to above that expansions of ceramic bodies similar in magnitude to those obtained during long periods of storage may be produced by subjecting them to steam at a pressure of 150 lbs./in.$^2$ for one hour in an autoclave. This treatment was, therefore, suggested as an accelerated test for indicating whether crazing from this expansion will develop in time on ceramic bodies.

[1] Research associates of the National Terra Cotta Society at the National Bureau of Standards.

[2] Methods for Testing Crazing of Glazes Caused by Increases in Size of Ceramic Bodies, J. Am. Cer. Soc., 11, pp. 271-277; 1928.

In general, this type of crazing is less apt to occur on vitreous than on porous bodies, although it may develop on some vitreous bodies. This led to the belief that it may depend upon the composition of the body as well as the degree of vitrification. Studies were made, therefore, to determine the effects of additions of different substances to ceramic bodies upon their expansions when treated with water or steam. It was found that the expansion of the body depends upon the mineral and chemical composition of the body as well as its absorption and therefore can be reduced and often prevented by proper manufacturing control. The results of the investigations warranting this conclusion are given below.

## II. EXPERIMENTAL METHODS

To determine the moisture expansions of the bodies, bars 24 inches long were fired to cone 6. This firing would drive off any combined water which might be present. Shortly after firing and cooling to 20° C. their lengths were measured by means of a microscopic comparator.[3] They were then completely immersed in water for 168 hours, after which their lengths were measured while wet and also after drying at 110° C. for 12 hours. The same specimens were then kept immersed under water for 240 and 312 hour periods, and their wet lengths were measured after each soaking period. They were dried at 110° C. for 20 hours, and their dry lengths at 20° C. were measured. The specimens were then boiled in water 4 hours, after which their wet and dry lengths were again measured. Their lengths were also determined at 20° C. subsequent to subjecting them to steam at pressures of 75, 100, and 125 lbs./in.$^2$ for 1 hour. All absorption tests were made by 5 hours, boiling in water.

## III. EFFECTS OF SILICA

The mixtures given in Table 1 were used to study the effects of silica on the moisture expansions of several types of ceramic bodies. The average percentages moisture expansion, measured and computed as the per cent linear expansion, in terms of the original dry lengths of the open-burning bodies with and without free silica added are shown in Figure 1. These values represent the averages obtained on two specimens in each case. It is apparent that the open-burning "pure clay"[4] body, which expanded considerably due to the action of water, expanded decidedly more after the addition of 40 per cent silica. Drying the specimens which have been immersed under water often increased their lengths. Heating the wet specimens during drying probably causes the water to react with the

[3] See footnote 2, p. 1.

[4] By "pure clay" is meant high-grade commercial clays, such as ball clays and kaolins, containing less than 5 per cent free silica.

body. After the specimens have been boiled or treated in an autoclave, however, they contract instead of expand upon drying. Why this obtains remains to be explained. From Figure 2 it can be seen that the addition of silica increased the moisture expansion of tight-burning pure clay bodies to a greater extent than it did open-burning pure clay bodies. For example, the tight-burning pure clay body developed practically no expansion when treated in the autoclave, but with 40 per cent silica it expanded about 0.065 per cent.

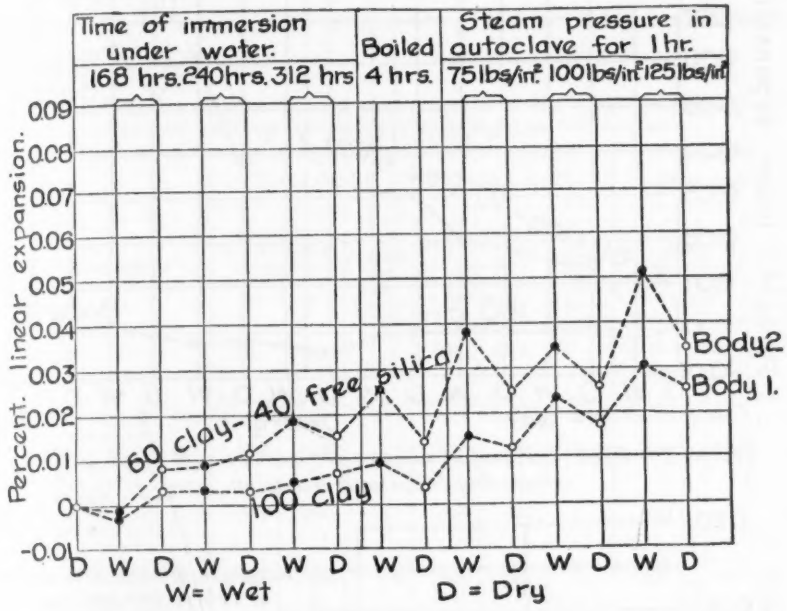


FIGURE 1.—*Effect of crystalline silica on changes in size of an open-burning pure clay due to water treatments*

TABLE 1.—*Composition of ceramic bodies containing varying amounts of clay and free silica*

| Body No.[1] | Bonding portion of body | | | | Composition of cone 6 grog | | | Clay and free silica in terms of total weight | | Absorption after 5 hours, boiling |
|---|---|---|---|---|---|---|---|---|---|---|
| | English ball clay | Florida kaolin | Potter's flint (quartzite) | Diatomaceous silica | English ball clay | Florida kaolin | Potter's flint (quartzite) | Clay | Free silica | |
| | | | | | | | | *Per cent* | *Per cent* | *Per cent* |
| 1 | 19.5 | 45.5 | | | 10.5 | 24.5 | | 100 | | 14.8 |
| 2 | 9.75 | 29.25 | 26.0 | | 5.25 | 15.75 | 14.0 | 60 | 40 | 19.6 |
| 3 | 65.0 | | | | 35.0 | | | 100 | | .7 |
| 4 | 39.0 | | 26.0 | | 21.0 | | 14.0 | 60 | 40 | 10.3 |
| 5 | 5.0 | | 95.0 | | | | | 5 | 95 | 16.8 |
| 6 | 20.0 | | 80.0 | | | | | 20 | 80 | 15.2 |
| 7 | 17.5 | | | 82.5 | | | | 17.5 | 82.5 | |

[1] The first 4 bodies will be designated as follows in this report: (1) Open-burning clay, (2) open-burning siliceous clay, (3) tight-burning clay, and (4) tight-burning siliceous clay.

It appears, therefore, that the tendency of ceramic bodies to develop moisture expansion may be decreased by selecting clays low in quartzite, which is generally the most common form of free silica

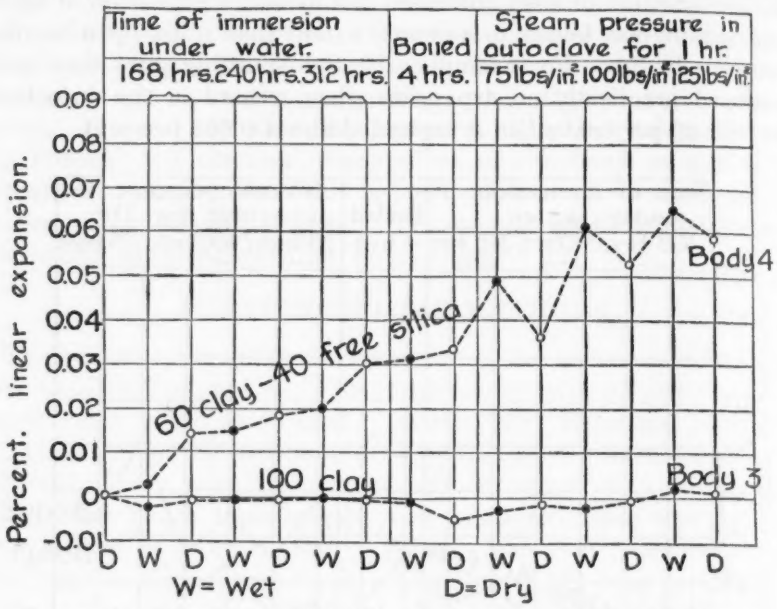


FIGURE 2.—*Effect of crystalline silica on changes in size of a tight-burning pure clay due to water treatments*

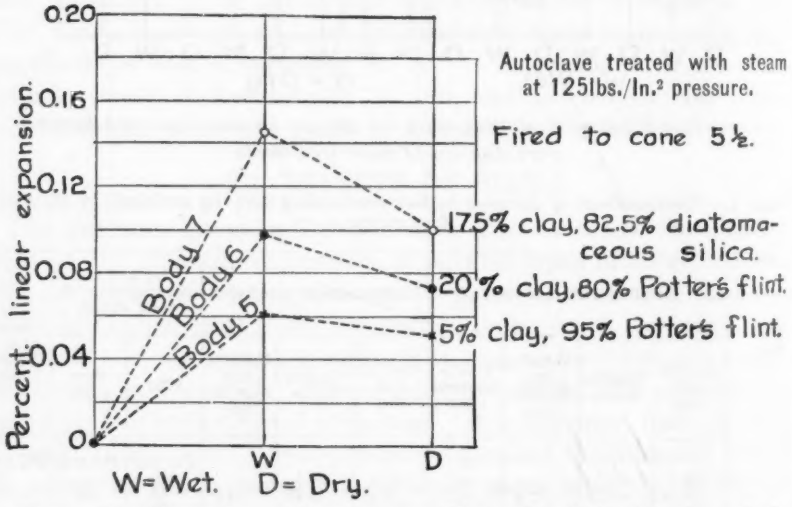


FIGURE 3.—*Expansions of diatomaceous silica and potter's flint when mixed with small percentages of clay*

in clays. It is evident from the results shown in Figure 3 that amorphous silica, such as diatomaceous silica, may increase the moisture expansion to a greater extent than quartzite, but this form

of silica is seldom found in large amounts in clays, such as are used for making burned clay products. This may be due to the fact that diatomaceous silica is more colloidal in nature than quartzite.

## IV. EFFECTS OF BLAST-FURNACE SLAG AND FELDSPAR

The effects of blast-furnace slag and feldspar additions on the moisture expansions of some bodies containing grog were determined after firing to different temperatures the bodies whose compositions are given in Table 2. The results are given in Table 2 and plotted in Figures 4 and 5.

TABLE 2.—*Moisture expansions and absorptions of bodies containing blast-furnace slag and feldspar after being fired to different temperatures*

| No. | Composition | Fired to cone 4 | | | Fired to cone 6 | | | Fired to cone 8 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Expansion after treatment in autoclave at 150 lbs./in.² for 1 hour | | Absorption after 5 hours boiling | Expansion after treatment in autoclave at 150 lbs./in.² for 1 hour | | Absorption after 5 hours boiling | Expansion after treatment in autoclave at 150 lbs./in.² for 1 hour | | Absorption after 5 hours boiling |
| | | Wet | Dry | | Wet | Dry | | Wet | Dry | |
| | | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* |
| A | 65 per cent clay,[1] 35 per cent grog. | 0.0895 | 0.0737 | 15.4 | 0.0574 | 0.0527 | 14.0 | 0.0464 | 0.0443 | 12.6 |
| B | 60 per cent clay, 22.5 per cent grog, 17.5 per cent slag, through 80 mesh. | .0182 | .0149 | 12.4 | .0099 | .0107 | 11.3 | .0052 | .0035 | 8.1 |
| C | 65 per cent clay, 17.5 per cent grog, 17.5 per cent slag, through 10 mesh. | .0339 | .0283 | 12.6 | .0124 | .0103 | 11.3 | .0070 | .0069 | 10.1 |
| D | 60 per cent clay, 22.5 per cent grog, 17.5 per cent feldspar, through 80 mesh. | .1204 | .0994 | 8.2 | .0464 | .0425 | 5.9 | .0430 | .0381 | 5.0 |
| E | 65 per cent clay, 17.5 per cent grog, 17.5 per cent feldspar, through 10 mesh. | .1119 | .0899 | 9.2 | .0594 | .0528 | 6.4 | .0540 | .0513 | 5.1 |

[1] The clay used in these bodies was a No. 2 fire clay, and the grog employed was calcined fire clay ground to pass through a 10-mesh sieve.

It was found that the addition of a blast-furnace slag to the body decreased its moisture expansion to a marked degree. The addition of feldspar in a few cases reduced this expansion only slightly, whereas in most cases it was increased. As a rule, the bodies containing 80-mesh as compared to 10-mesh feldspar showed lower expansion. More intimate mixtures can be obtained with finely ground material. The reverse is true with low-fired bodies, the reason for which remains to be determined.

Increasing the firing temperature of all bodies from cone 4 to cones 6 and 8 and thereby reducing their absorptions also reduced the moisture expansions. There is, however, apparently no direct relation (fig. 5) between absorptions and moisture expansions of bodies of

different compositions. For example, bodies containing feldspar have comparatively low absorptions but develop relatively high moisture expansions, whereas bodies with blast-furnace slag may have much

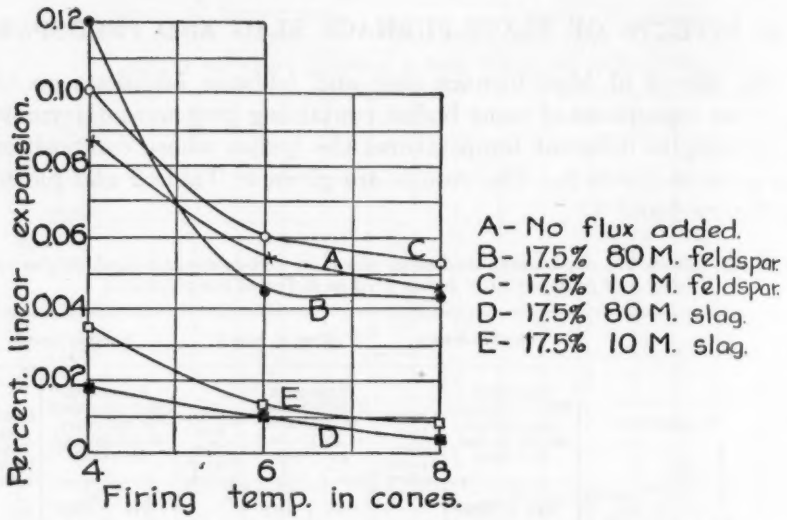


FIGURE 4.—*The effects of feldspar and slag additions to a clay body on its expansion due to autoclave treatment after firing to different temperatures*

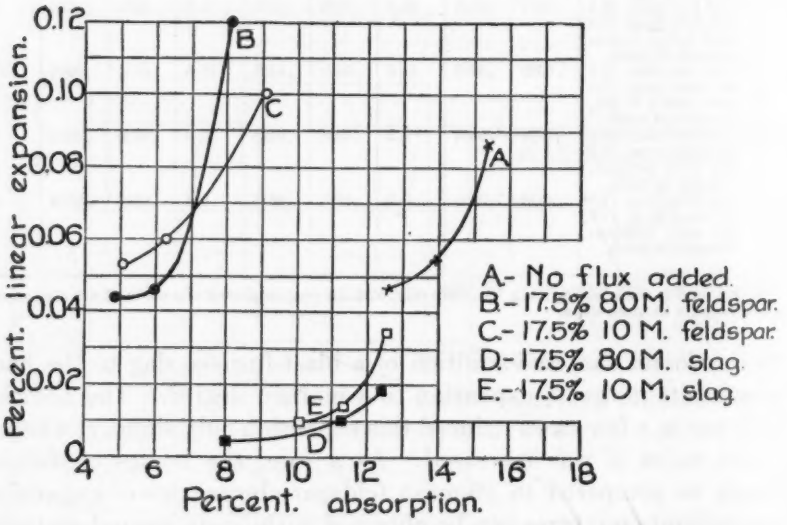


FIGURE 5.—*The relation between absorptions and expansions of some bodies due to autoclave treatments*

higher absorptions but still develop lower moisture expansions. For bodies having the same compositions, however, the moisture expansion increases with an increase in absorption.

## V. MOISTURE EXPANSIONS OF CLAY, FELDSPAR, AND FLINT MIXTURES

The moisture expansions of the mixtures given in Table 3 containing varying amounts of clay, feldspar, and potter's flint were determined after immersing the specimens in water 40 hours and boiling in water 2 hours, and also after treating them in an autoclave at 150 lbs./in.$^2$ steam pressure for 1 hour. The results are given in Table 3 and are plotted in Figures 6 and 7.

TABLE 3.—*The moisture expansions of clay, feldspar, and flint mixtures after having been fired to cone 5½*

| Body No. | Composition | | | Expansion | | | Absorption after 5 hours' boiling |
|---|---|---|---|---|---|---|---|
| | Tennessee No. 5 ball clay | Maine feldspar | Potter's flint quartzite | Soaked in water 40 hours, boiled 2 hours, and dried at 110° C. | Treated in an autoclave at 150 lbs./in.$^2$ steam pressure for 1 hour | | |
| | | | | | Wet | Dry | |
| | | | | *Per cent* | *Per cent* | *Per cent* | *Per cent* |
| 1 | 100 | | | 0.032 | 0.109 | 0.063 | 5.44 |
| 2 | 80 | 20 | | .040 | .054 | .067 | 1.51 |
| 3 | 80 | | 20 | .014 | .115 | .078 | 8.86 |
| 4 | 60 | 40 | | −.002 | .004 | .000 | .38 |
| 5 | 60 | 20 | 20 | .016 | .066 | .062 | 3.87 |
| 6 | 60 | | 40 | .008 | .104 | .066 | 13.45 |
| 7 | 40 | 60 | | −.004 | .002 | −.008 | .29 |
| 8 | 40 | 40 | 20 | .011 | .023 | .010 | .46 |
| 9 | 40 | 20 | 40 | .002 | .061 | .051 | 3.52 |
| 10 | 40 | | 60 | .011 | .081 | | 3.28 |
| 11 | 20 | 80 | | .004 | .007 | .005 | .18 |
| 12 | 20 | 60 | 20 | .010 | .025 | .021 | .20 |
| 13 | 20 | 40 | 40 | .015 | .051 | .043 | .97 |
| 14 | 20 | 20 | 60 | .016 | .096 | .088 | 5.68 |
| 15 | 20 | | 80 | .035 | .097 | .073 | 15.22 |
| 16 | 5 | 95 | | −.009 | .004 | .000 | .15 |
| 17 | | 80 | 20 | .011 | .059 | .058 | .41 |
| 18 | | 60 | 40 | .011 | .100 | .079 | 4.98 |
| 19 | | 40 | 60 | .040 | .188 | .162 | 13.44 |
| 20 | 5 | | 95 | .016 | .061 | .051 | 16.80 |
| 21 | | 85 | 15 | .010 | .105 | .090 | 2.70 |

It is evident from the results plotted in Figure 6 that the addition of feldspar to "pure clay" materially reduces the moisture expansion of the fired body, whereas the addition of feldspar to siliceous clay reduces the moisture expansion only slightly. They also indicate that a mixture of feldspar and silica may develop a greater moisture expansion than either feldspar or flint alone. This expansion was found to be greatest with a mixture composed approximately of 40 per cent feldspar and 60 per cent flint, indicating a reaction between the flint and silica.

The relations between absorptions and moisture expansions of the mixtures are shown in Figure 7. The moisture expansions of all mixtures with high absorptions are usually large. Of the mixtures with low absorptions, those composed of "pure clay" and feldspar

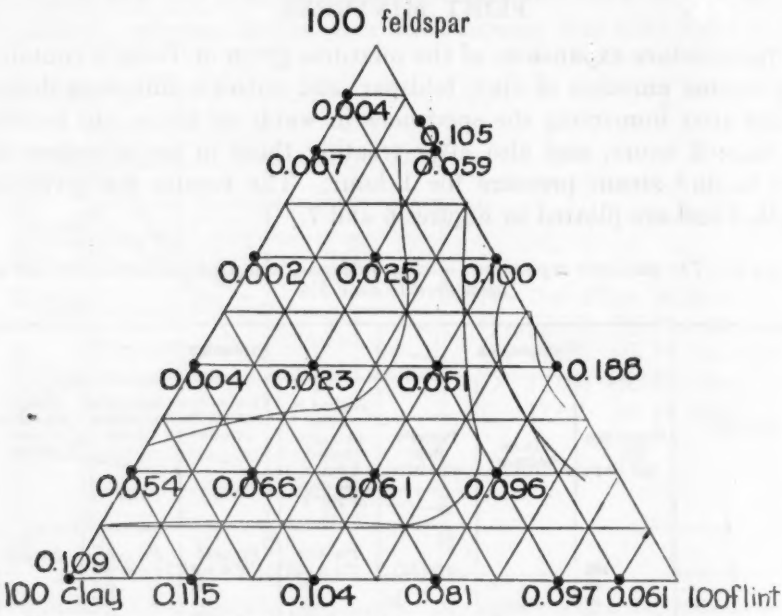


FIGURE 6.—*Expansions of clay-feldspar-silica mixtures due to autoclave treatments*

(Specimens were fired to cone 5½)

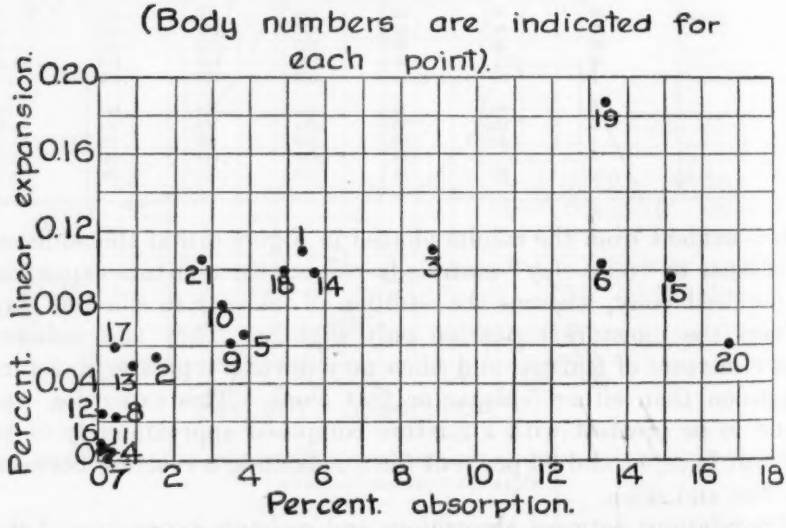


FIGURE 7.—*Relations between the absorptions of clay-feldspar-silica and their expansions due to autoclave treatments*

have developed practically no expansion, whereas those composed of clay, silica, and feldspar have developed considerable moisture expansion. Note the difference between clay, silica, and feldspar mixtures, clay, and silica, and clay and feldspar, two component mixtures. (See fig. 6.)

It must be remembered that these bodies were fired to cone 5½ and different results would be obtained if they were fired to different temperatures.

## VI. EFFECTS OF IRON OXIDE, CALCIUM CARBONATE, MAGNESIUM CARBONATE, AND TITANIUM OXIDE

The mixtures given in Table 4 were studied to determine the effects of iron oxide, calcium carbonate, magnesium carbonate, and titanium oxide upon the moisture expansions of ceramic bodies. Sufficient magnesium and calcium carbonates were added, as magnesium and whiting, respectively, to the bodies to introduce the equivalents of 5 per cent, respectively, of magnesium and calcium oxides.

It is apparent from the results shown in Table 4 and Figure 8 that finely ground magnesite was most effective for reducing the moisture expansion of this body, since the addition of 10.5 per cent reduced the expansion from 0.097 to 0.011 per cent.

TABLE 4.—*The expansions caused by water treatments of bodies containing whiting, magnesite, iron oxide, and titanium oxide*

[Specimens were fired to cone 7]

| No. | Additions to a body composed of 75 per cent Tennessee No. 9 ball clay and 25 per cent grog | Expansion caused by immersing in water 168 hours | | Expansion caused by treatment in an autoclave at 150 lbs./in.² pressure for 1 hour | | Absorption after 5 hours' boiling |
|---|---|---|---|---|---|---|
| | | Wet | Dry | Wet | Dry | |
| | | *Per cent* | *Per cent* | *Per cent* | *Per cent* | *Per cent* |
| 1 | Body with no additions | 0.008 | 0.017 | 0.097 | 0.073 | 10.1 |
| 2 | 8.95 per cent calcium carbonate through a 200-mesh sieve | .001 | .007 | .045 | .042 | 11.8 |
| 3 | 10.5 per cent magnesium carbonate through a 200-mesh sieve | .001 | .006 | .011 | .009 | 8.9 |
| 4 | 3 per cent iron oxide | .002 | .012 | .040 | .032 | 7.5 |
| 5 | 5 per cent titanium oxide | .006 | .018 | .079 | .062 | 9.1 |

Three per cent iron oxide reduced the expansion caused by autoclave treatment to 0.040 per cent, which is not so effective as the addition of 10.5 per cent magnesite to the body. Adding 8.95 per cent whiting to the body reduced the moisture expansion caused by autoclave treatment to 0.045 per cent. It is also not so effective as magnesite. Adding 5.0 per cent titanium oxide to the body decreased the expansion to 0.079 per cent, being less effective than the other fluxes used.

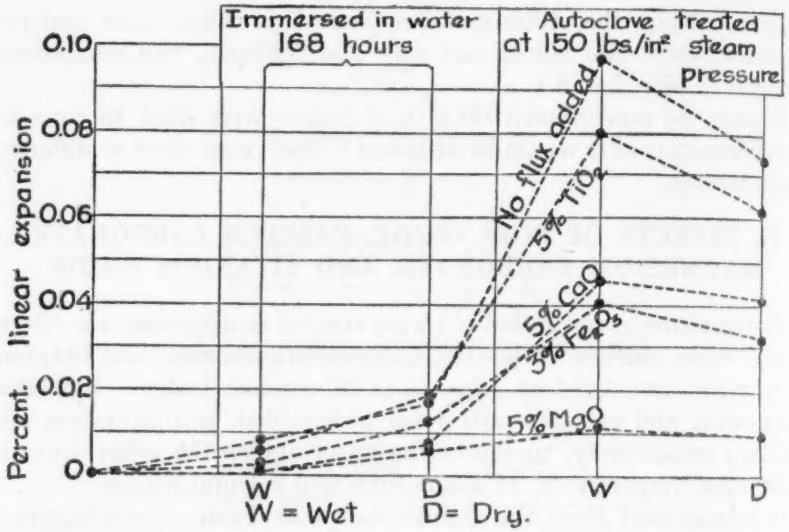


FIGURE 8.—*Effects of $TiO_2$, $CaO$, $Fe_2O_3$, and $MgO$ additions on the tendency of clay bodies to expand from water treatments*

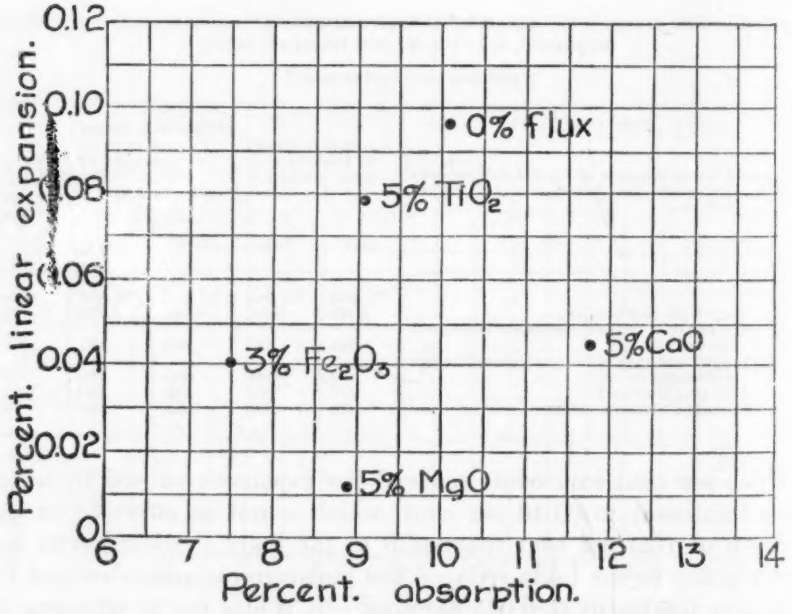


FIGURE 9.—*The relation between absorptions and expansions of bodies containing $CaO$, $MgO$, $Fe_2O_3$, and $TiO_2$*

The relation between absorptions and moisture expansions of these mixtures is shown in Figure 9. It is evident that the body containing 5 per cent magnesium oxide has a low moisture expansion in spite of the fact that it has a comparatively high absorption, whereas bodies containing the other fluxes have developed relatively high moisture expansions.

It seems evident from these results that by using finely-ground magnesite or dolomite as a flux in place of whiting in ceramic bodies that their resistance to moisture expansion can be improved.

## VII. SUMMARY AND CONCLUSIONS

1. Fired mixtures of "pure clay" and feldspar, with low absorptions, showed comparatively low moisture expansions when subjected to steam at 150 lbs./in.$^2$ pressure for one hour. Similar mixtures with low absorptions, but containing 20 per cent free silica, however, expanded considerably.

2. With the exception of bodies containing slag or magnesite, all others tested having high absorptions developed considerable moisture expansions.

3. The moisture expansion of a siliceous fire-clay body was decreased by the addition of blast-furnace slag, while the expansion was increased in some cases and reduced only slightly in others by the addition of feldspar.

4. By adding finely-ground magnesite to a ball-clay body, the moisture expansion was reduced more than by the addition of whiting, iron oxide, or titanium oxide. The magnesite used was ground to pass a 200-mesh sieve in this case. Coarsely ground magnesite may not be as beneficial as this finely-ground material.

5. Increasing the firing temperature of ceramic bodies reduces their moisture expansion to a marked extent in most cases.

WASHINGTON, March 18, 1929.